Chapter # 3 DC Machines

1. Introduction

Applications such as heaters require energy in electrical form. In other applications, such as fans, energy is required in mechanical form. One form of energy can be obtained from the other form with the help of converters. Converters that are used to continuously translate electrical input to mechanical output or vice versa are called *electric machines*. The process of translation is known as *electromechanical energy conversion*. An electric machine is therefore a link between an electrical system and a mechanical system, as shown in Fig. 1.

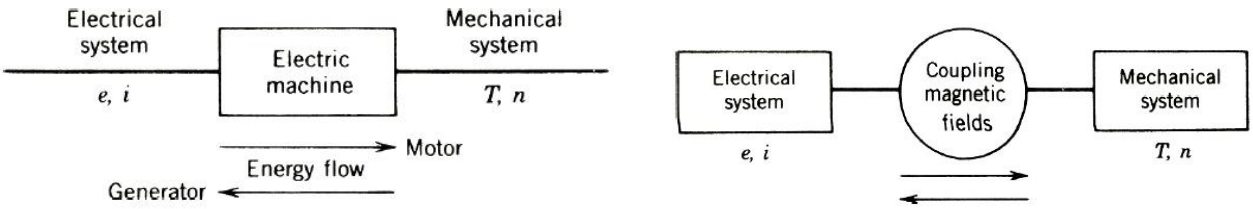


Fig. 1, Electromechanical energy conversion

There are three types of electrical machines: DC, Induction, Synchronous.

These types are based on the following two electromagnetic phenomena:

- When a conductor moves in a magnetic field, voltage is induced in the conductor. (generating actions) (Right-hand Rule)

- When a current-carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. (motoring actions) (Left-hand Rule)

In case of generating action, an expression for the voltage induced in a conductor moving in a magnetic field can be derived. As shown in Fig. 2, if a conductor of length *l* moves at a linear speed *v* in a magnetic field *B*, the voltage in the conductor is:

$$e_{ind} = B \times l \times v$$

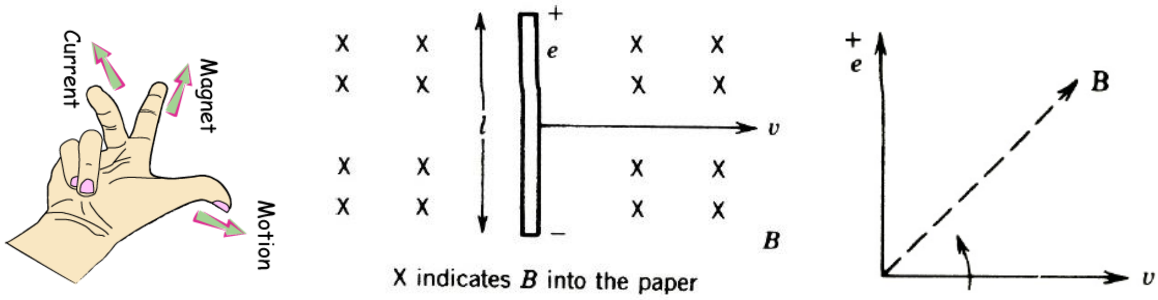


Fig. 2, Generating action

Example:

Figure 3 shows a conductor moving with a velocity of 5.0 m/s to the right in the presence of a magnetic field. The flux density is 0.5 T into the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

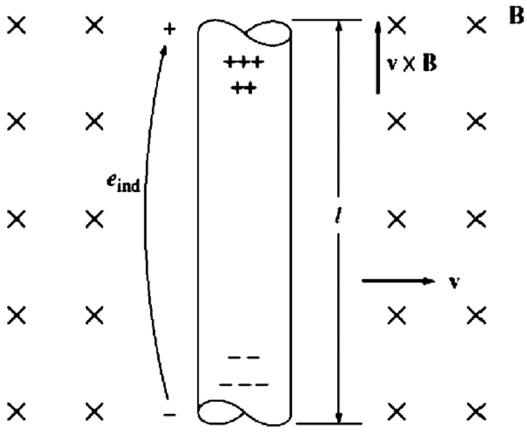


Fig. 3, A conductor moving in the presence of a magnetic field.

$e_{ind} = B\ l\ v = 0.5\ \times 1\ \times 5 = 2.5\ V$ Polarity as shown in figure

Example:

Figure 4 shows a conductor moving with a velocity of 10 m/s to the right in a magnetic field. The flux density is 0.5 T, out of the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the induced voltage?

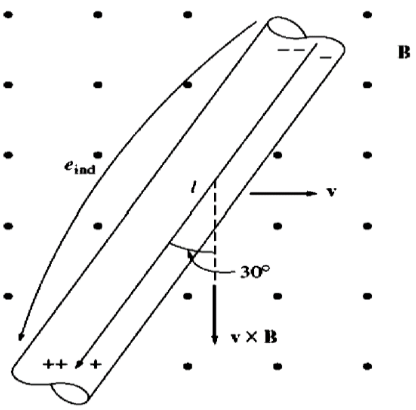


Fig. 4, A conductor moving in the presence of a magnetic field.

$e_{ind} = B\ l\ v = 0.5\ \times (1 \times \cos(30)) \times 10 = 4.33\ V$ Polarity as shown in figure

For the current-carrying conductor shown in Fig. 5, the force produced on the conductor is

$$F = B\ \times\ l\ \times\ i$$

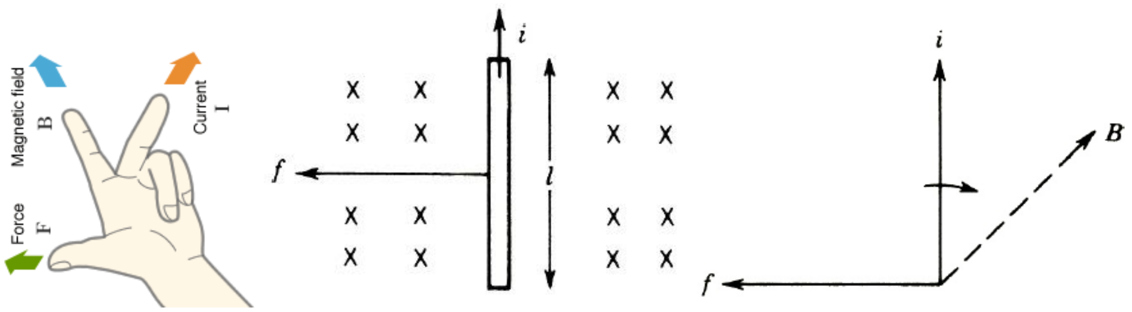


Fig. 5, Motor action

Example:

Figure 6 shows a wire carrying a current in the presence of a magnetic field. The magnetic flux density is 0.25 T. directed into the page. If the wire is 1.0 m long and carries 0.5 A in the direction from top of bottom of the page. what are the magnitude and direction of the force induced on the wire?

$$F = B\ \times\ l\ \times\ i = 0.25\ \times 1\ \times 0.5 = 0.125\ N$$

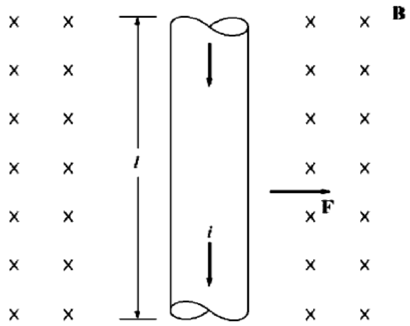


Fig. 6, A current-carrying wire in the presence of a magnetic field

1.1 Basic difference between AC and DC Machines

DC motor has an armature (that carry the conductors) and rotate (Rotor), but the magnetic field winding stays stand still (Stator). Direct current is supplied to rotating armature by means of Brush-Commutator arrangement. Field winding may be in series or shunt with armature and fed with dc supply to create flux. So, DC supply goes to both armature and field winding.

On the other hand, AC (Induction) motor has an armature that stays stand still (Stator), but the rotor that rotates provide the magnetic field. Only AC is supplied to stator which generates rotating magnetic field, then by Faraday's laws of electromagnetic induction this rotating magnetic field induce EMF in rotor circuit, thus produce current in it.

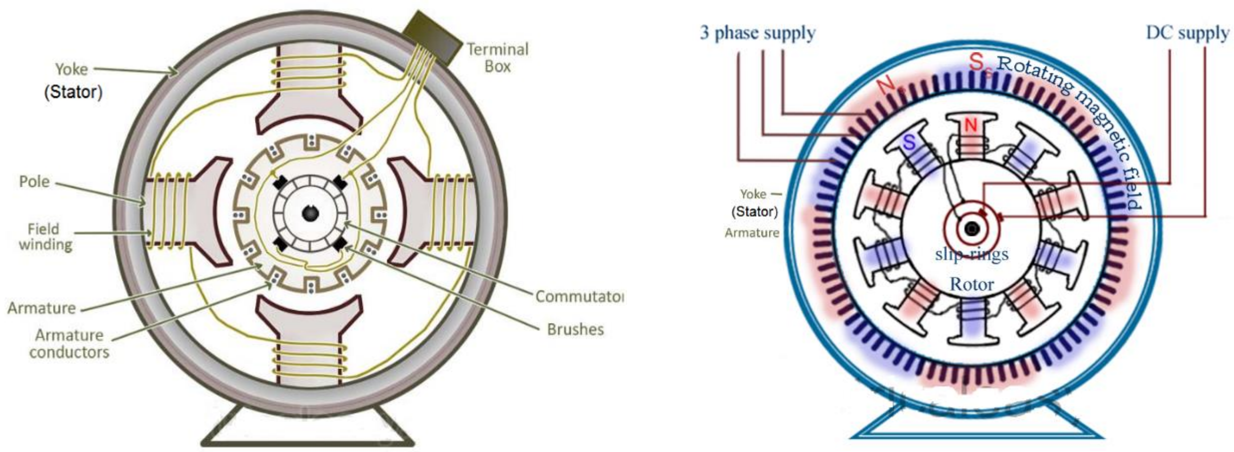
2. Basic Structure of Electric Machines

The structure of an electric machine has two major components, stator and rotor, separated by the air gap, as shown in Fig. 7.

Stator: This part does not move and normally is the outer frame of the machine.

Rotor: This part is free to move and normally is the inner part of the machine.

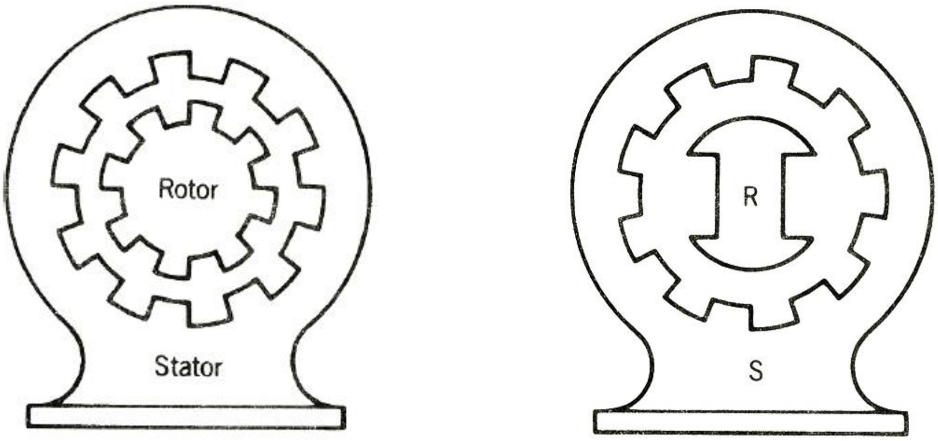
Air gap: is a small clearance between stator and rotor to allow the later to rotate freely.



a) DC machines b) AC machines

Fig. 7, Stator and rotor for different machines

Both stator and rotor are made of ferromagnetic materials. In most machines, slots are cut on the inner periphery of the stator and outer periphery of the rotor structure, as shown in Fig. 8. Copper conductors are placed in these slots.



Cylindrical Salient

Fig. 8, Structure of electric machines.

Although a dc machine can operate as either a generator or a motor, at present its use as a generator is limited because of the widespread use of ac power. The dc machine is extensively used as a motor in industry. Its speed can be controlled over a wide range with relative ease. Large dc motors (in tens or hundreds of horsepower) are used in machine tools, printing press, conveyors, fans, pumps, textile mills, and so on. Additionally, dc motors still dominate as traction motors used in transit cars and locomotives. Small dc machines (in fractional horsepower rating) are used primarily as control devices, such as tachogenerators for speed sensing and servomotors for positioning and tracking. The dc machine definitely plays an important role in industry.

2.1 One-Coil Operation:

If the coil is assumed to rotate in clock-wise direction as shown in Fig. 9, the flux linked with the coil changes. Hence, an e.m.f. is induced in it which is proportional to the rate of change of flux linkages ($e = N\, d\Phi/dt$). When the plane of the coil is at right angles to lines of flux *i.e.* when it is in position, 1, then flux linked with the coil is maximum but rate of change of flux linkages is minimum. It is so because in this position, the coil sides *AB* and *CD* do not cut or shear the flux, rather they slide along them *i.e.* they move parallel to them. Hence, there is no induced e.m.f. in the coil. Let us take this no-e.m.f. or vertical position of the coil as the starting position. The angle of rotation or time will be measured from this position.

As the coil continues rotating further, the rate of change of flux linkages (and hence induced e.m.f. in it) increases, till position 3 is reached where θ = 90º. Here, the coil plane is horizontal *i.e.* parallel to the lines of flux. As seen, the flux linked with the coil is minimum but rate of change of flux linkages is maximum. Hence, maximum e.m.f. is induced in the coil when in this position.

In the next quarter revolution *i.e.* from 90º to 180º, the flux linked with the coil gradually increases but the rate of change of flux linkages decreases. Hence, the induced e.m.f. decreases gradually till in position 5 of the coil, it is reduced to zero value.

In the next half revolution *i.e.* from 180º to 360º, the variations in the magnitude of e.m.f. are similar to those in the first half revolution. Its value is maximum when coil is in position 7 and minimum when in position 1. But it will be found that the direction of the induced current is from *D* to *C* and *B* to *A*

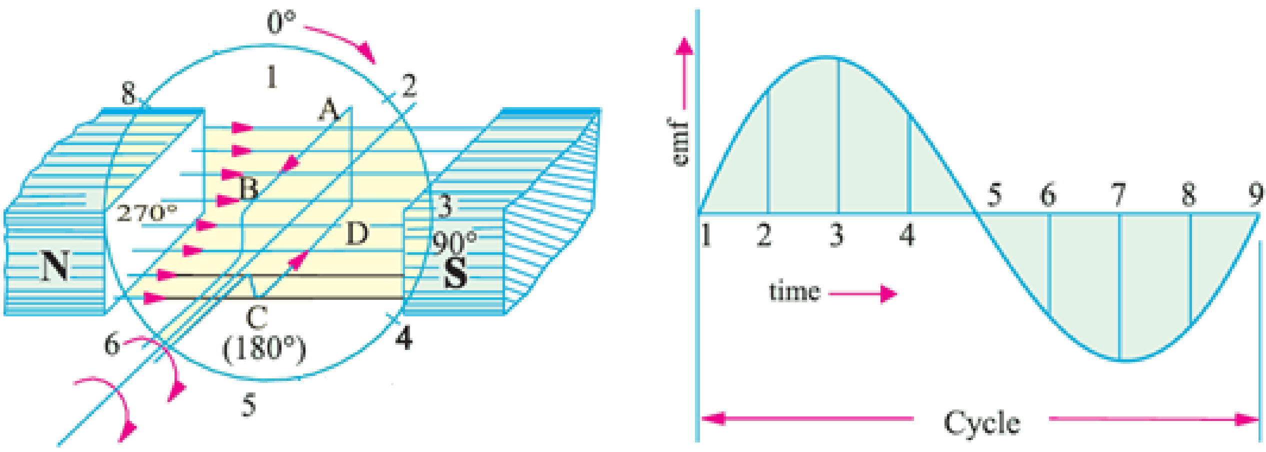


Fig. 9, One-coil operation

It is clear that the current obtained from this simple generator reverses its direction after every half revolution. Such a current undergoing periodic reversal is known as alternating current.

For making the flow of current unidirectional in the external circuit, a conducting cylinder which is cut into two halves (segments) insulated from each other by a thin sheet of mica are used as shown in Fig. 10. The coil ends are joined to segments *a* and *b* on which rest the carbon brushes.

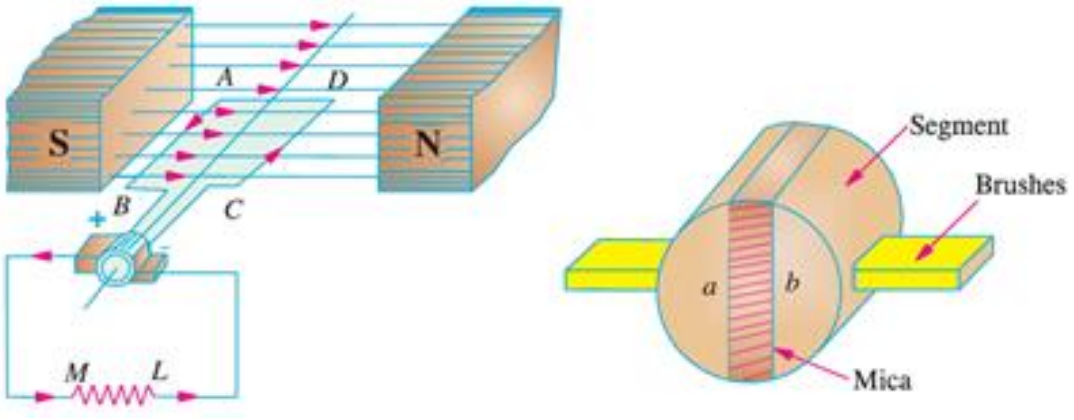


Fig. 10, One-coil machine with two segments

It is seen from Fig. 11 that in the first half revolution current flows along *(ABMLCD)* *i.e.* the brush No. 1 in contact with segment **'*a*'** acts as the positive end of the supply and **'*b*'** as the negative end. In the next half revolution, the direction of the induced current in the coil has reversed (*DCMLBA*). But at the same time, the positions of segments **'*a*'** and **'*b*'** have also reversed with the result that brush No. 1 comes in touch with the segment which is positive *i.e.* segment **'*b*'** in this case. Hence, current in the load resistance again flows from ***M*** to ***L***. This current is unidirectional but not continuous like pure direct current. The rectifying action is done using the split-rings (also called commutator).

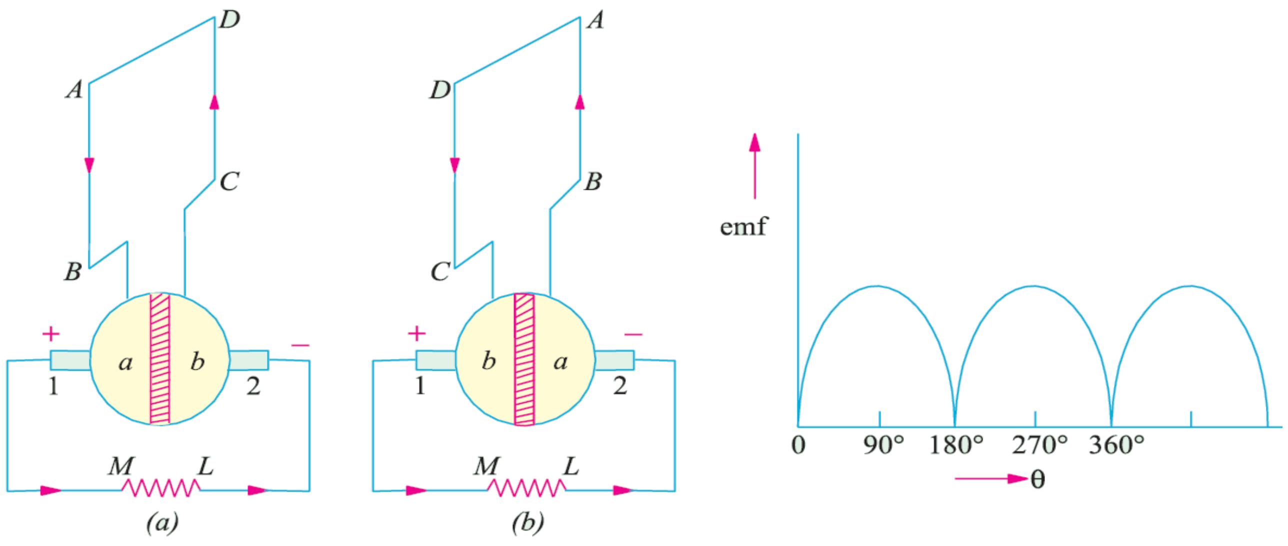
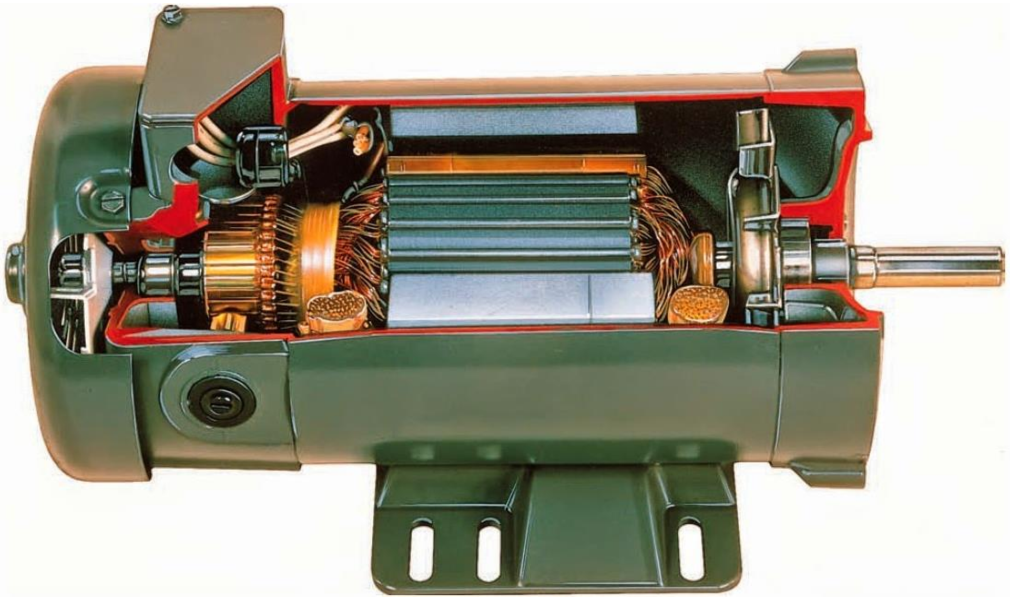
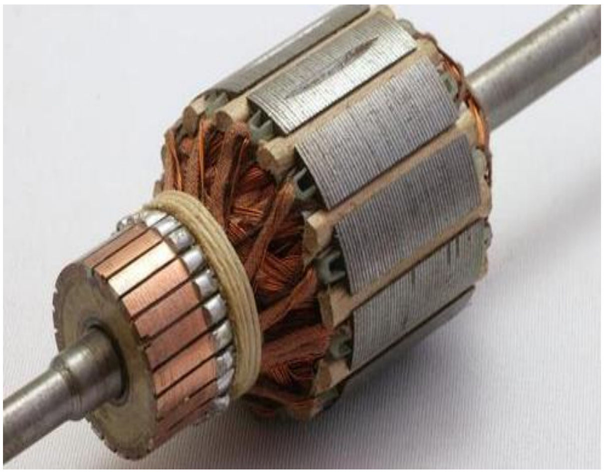


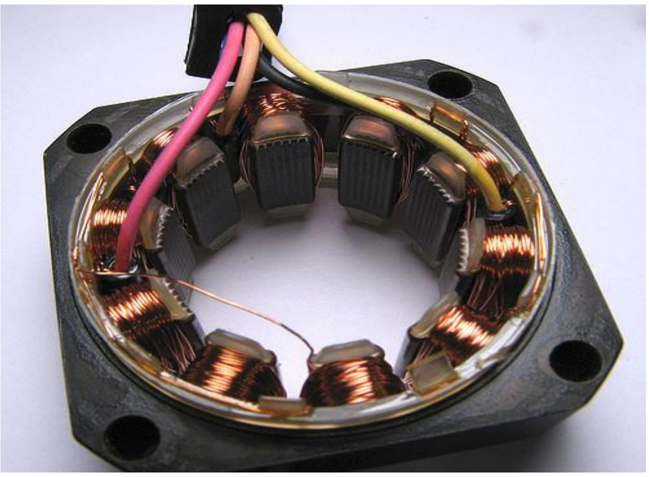
Fig. 11, Rectification of load current

2.2 Construction of DC Machines:

The construction of practical DC machines is shown in Fig. 12.







Rotor Stator

Fig. 12, DC machine layout

Usually, DC machines are consisting of the following essential parts:

1. Magnetic Frame or Yoke or stator

2. Pole-Cores and Pole-Shoes

3. Exciting Coils or Field Coils

4. Armature Core or rotor

5. Commutator

6. Brushes and Bearings

7. Armature Windings or Conductors

Stator or Yoke:

The outer frame or yoke shown in Fig. 13, serves double purpose; It provides mechanical support for the poles and acts as a protecting cover for the whole machine. It also carries the magnetic flux produced by the poles.

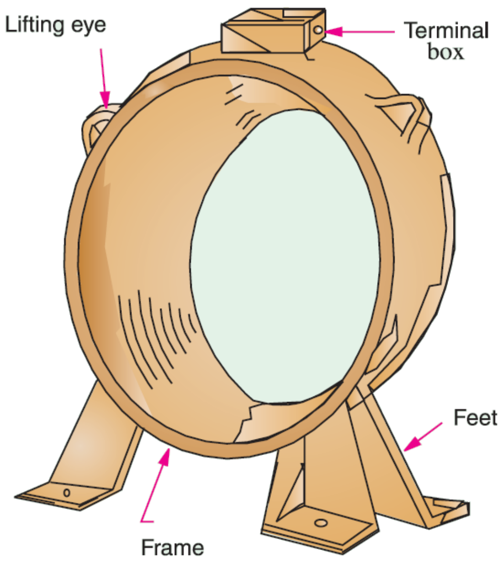


Fig. 13, Stator of DC machines

In small generators where cheapness rather than weight is the main consideration, yokes are made of cast iron. But for large machines usually cast steel or rolled steel is employed.

Pole-Cores and Pole-Shoes:

The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes; They spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path. They also support the field coils as shown in Fig. 14.

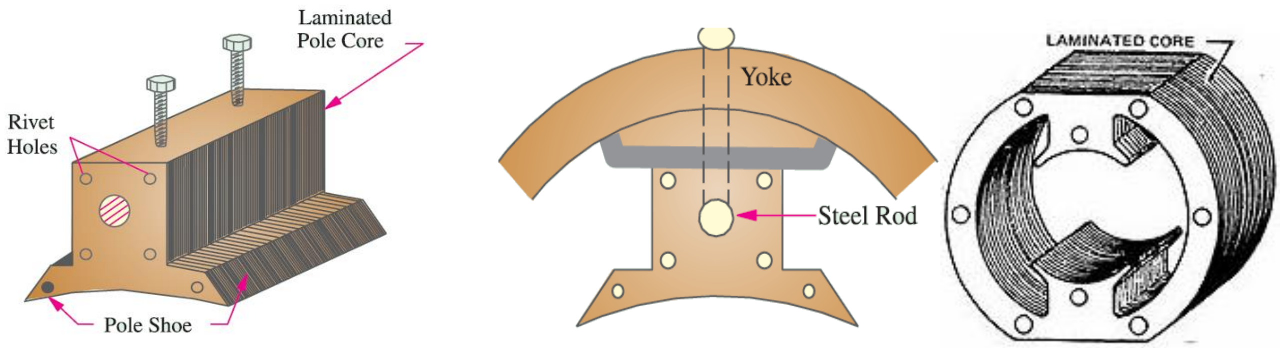


Fig. 14, Pole core and pole shoe

Exciting Coils or Field Coils:

The field coils, which consist of copper wire, are former-wound for the correct dimension. Then, the former is removed, and wound coil is put into place over the core as shown in Fig. 15. When current is passed through these coils, they electromagnetise the poles which produce the necessary flux that is cut by revolving armature conductors.

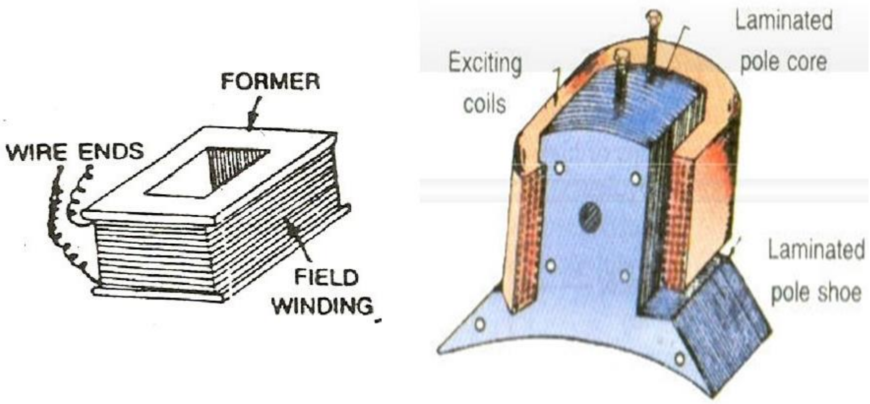


Fig. 15 Field coils

Armature Core:

It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets. In addition to this, its most important function is to provide a path of very low reluctance to the flux through the armature from a *N*-pole to a *S*-pole. It is cylindrical shaped and is built up of usually circular laminations approximately 0.5 mm thick as shown in Fig. 16. It is keyed to the shaft. Usually, these laminations include holes for air ducts which permits axial flow of air through the armature for cooling purposes.

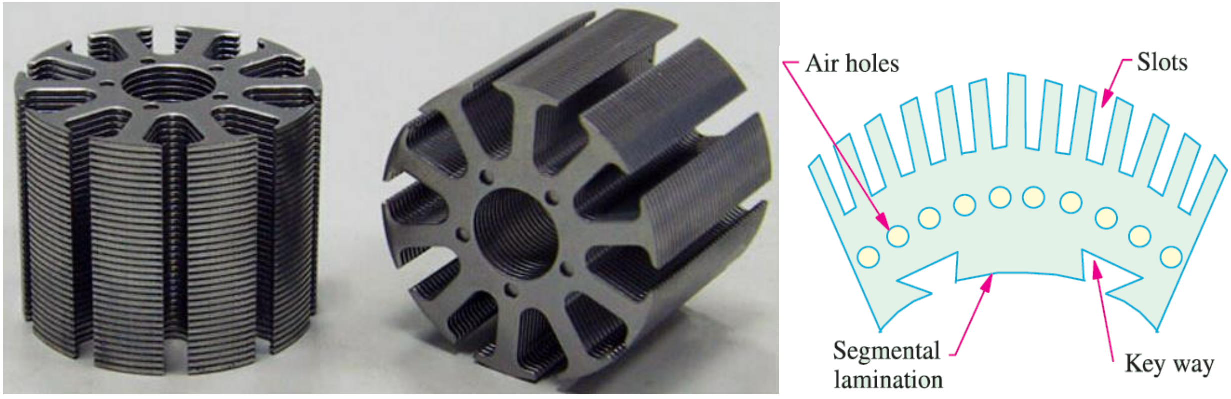


Fig. 16 Lamination core of armature

Commutator:

The function of the commutator is to facilitate collection of current from the armature conductors. It converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit. It is of copper cylindrical structure and is built up of wedge-shaped segments. These segments are insulated from each other by thin layers of mica. The number of segments is equal to the number of armature coils. Each commutator segment is connected to the armature conductor.

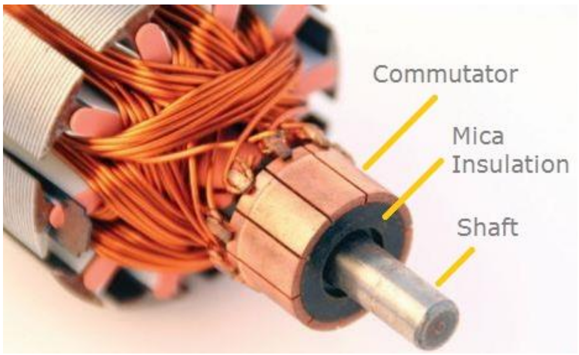


Fig. 17, Commutator structure

Brushes and Bearings

The function of brushes is to collect current from commutator. They are usually made of carbon or graphite and its shape is rectangular. These brushes are housed in brush-holders usually of the box-type variety. As shown in Fig. 18, the brushes are made to bear down on the commutator by a spring.

Roller bearings are used for quiet operation and to reduced bearing wear.

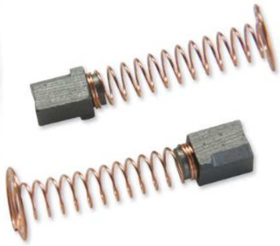


Fig. 18, Bearings and brushes

Magnetic circuit of DC Machines:

In DC machines, there are as many magnetic paths as the number of poles. For example, there are 4 magnetic paths for the DC machine shown in Fig. 19, equal the number of poles. Each complete path includes yoke, pole core, pole shoe, air gaps (stator to rotor & rotor to stator), armature teeth, armature core, pole shoe of adjacent pole, pole core of adjacent pole, yoke.

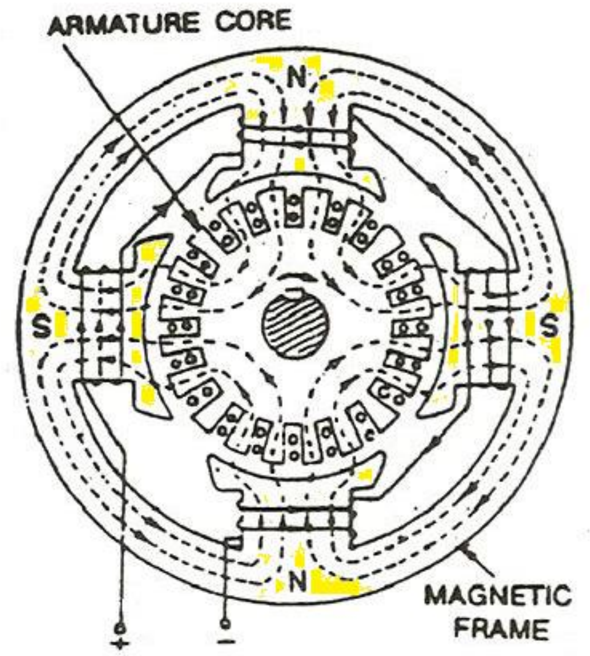


Fig. 19, Magnetic path in DC machines

The flux of N pole divided into 2 halves going to the neighbouring S poles. Similarly, the flux entering S pole is formed by two portions: half coming from neighbouring N poles.

There is some flux leaking from one pole to adjacent pole without entering the armature. The effective value of flux in poles is greater than that in the air gap. Yoke should carry the useful flux and leakage flux.

Assuming that A_p is the c.s.a of pole, A_g is the c.s.a of air gap, A_t is the c.s.a of armature teeth, A_c is the c.s.a of armature core, A_y is the c.s.a of yoke. Thus the corresponding values of the flux density for the useful flux (Φ) are:

$$B_p = \frac{\Phi}{A_p}, B_g = \frac{\Phi}{A_g}, B_t = \frac{\Phi}{A_t}, B_c = \frac{\Phi}{A_c}, B_y = \frac{\Phi}{A_y}$$

The MMF required per unit length (H) for various parts is obtained by using reference magnetization curves for the yoke and armature materials. If the permeability is known, we can calculate H as follows:

$$H = \frac{B}{\mu}$$

Therefore, we can calculate H_p, H_g, H_t, H_c and H_y.

The total value of MMF is then:

$$MMF = H_p l_p + H_g l_g + H_t l_t + H_c l_c + H_y l_y$$

Armature windings:

Let us define some basic components of the armature winding and terms as shown in Fig. 20:

- A turn consists of two conductors.
- A coil is formed by connecting several turns in series, with two ends (S=start & F=finish). Therefore, each coil has two coil sides.
- A winding is formed by connecting several coils in series.

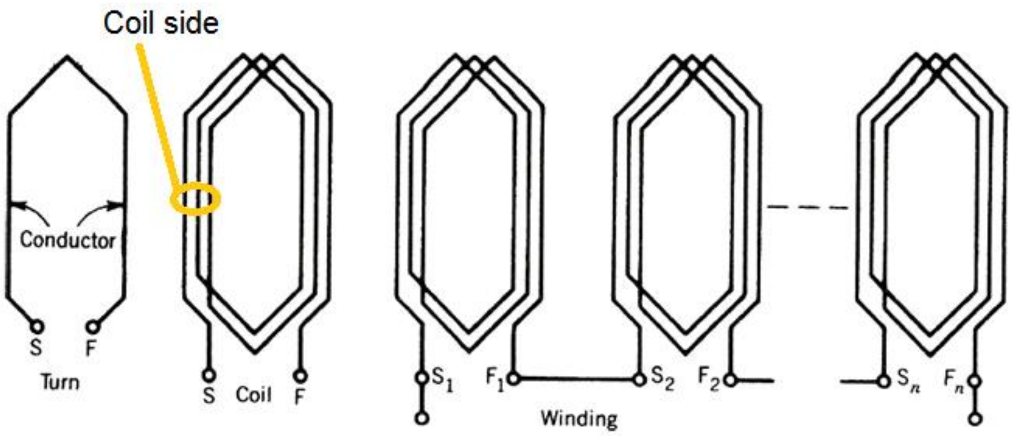


Fig. 20, Turn, coil, and winding

Consider a stator of a DC machine with four poles (2P=4) shown in Fig. 21. Therefore, in going around the air gap once (i.e., one mechanical cycle 2π that is called mechanical angle θ_m), two cycles of variation of the flux density distribution are encountered (that is called electrical angle θ_e)

$$\theta_e = P\,\theta_m$$

Pole Pitch:

The distance between the centers of two adjacent poles is known as pole pitch (τ)

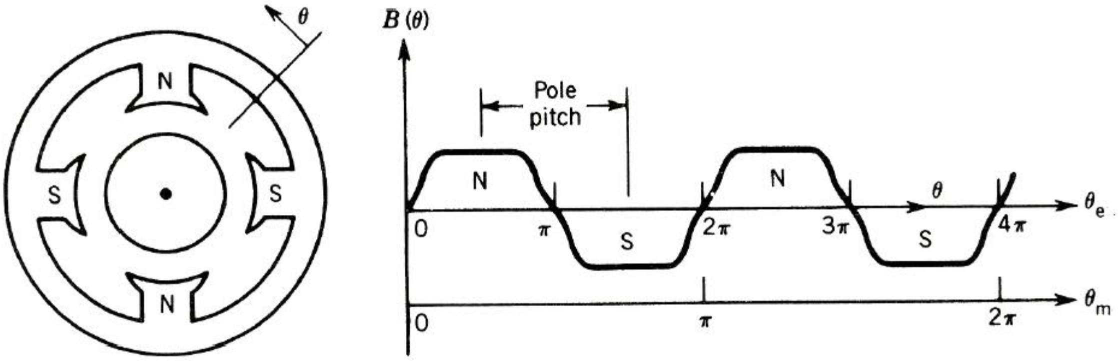


Fig. 21, Pole pitch

From another point of view, the pole pitch can be defined by distance measured in terms of armature slots. This done by dividing the total number of slots (S) by the total number of poles (2P)

$$\tau = \frac{S}{2P}$$

Coil Pitch or Coil Span:

The two sides of a coil are placed in two different slots on the rotor surface. The distance between the two sides of a coil is called the coil pitch. If the coil pitch is equal pole pitch (distance between coil sides is 180° electrical), it is called a full-pitch coil. If the coil pitch is less than one pole pitch, the coil is known as a short-pitch (or fractional pitch) coil. Short-pitch coils are desirable in ac machines for various reasons. The dc armature winding is made of full-pitch coils. Also, if the quotient of S and 2P is not integer, fractional pitch is used. Figure 22 indicates the two types of the coil pitch.

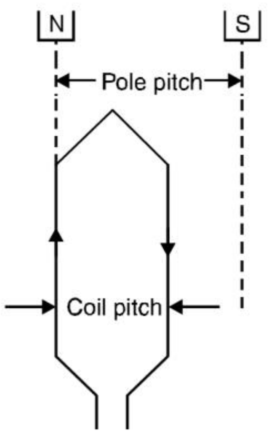


Fig. 22, a) Full pitch b) Fractional pitch

Single and double-layer windings:

Single-layer: It is that winding in which one coil side is placed in each armature slot. Such a winding is not much used.

Double-layer: In this type of winding, there are two coil sides per slot arranged in two layers. The coil side of one coil is placed in the upper layer of one slot (represented by solid line), while the other coil side is placed in the lower layer (represented by dashed line) of another slot (Fig. 23).

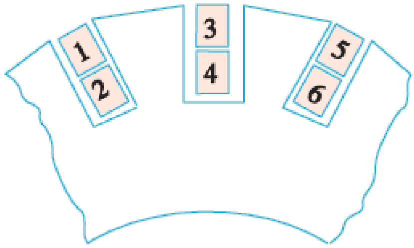


Fig. 23, Double-layer windings

Example:

The armature of a DC generator has 10 slots. Calculate the coil pitch in following cases:

a) Two-pole winding b) Four-pole winding

$$\tau = \frac{S}{2P} = \frac{10}{2} = 5$$

$$\tau = \frac{S}{2P} = \frac{10}{4} = 2.5$$

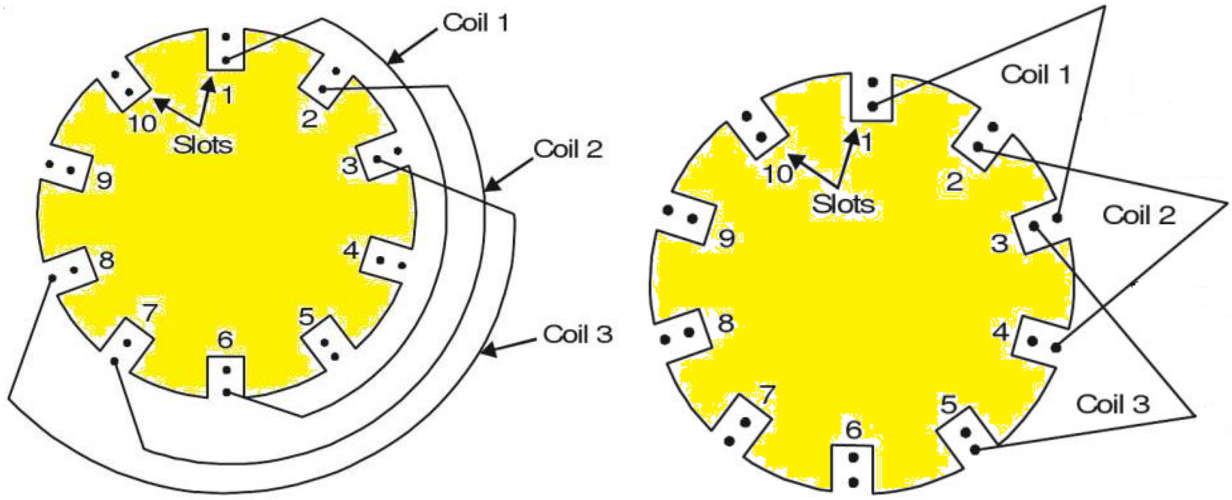


Fig. 24, Case (a) full pitch Case (b) fractional pitch

Note that, starting of the coil is at upper layer and ends at lower layer.

There are many ways in which the coils of the armature windings of a dc machine can be interconnected;

a) Progressive
b) Retrogressive, (rarely used), as illustrated in Fig. 25.

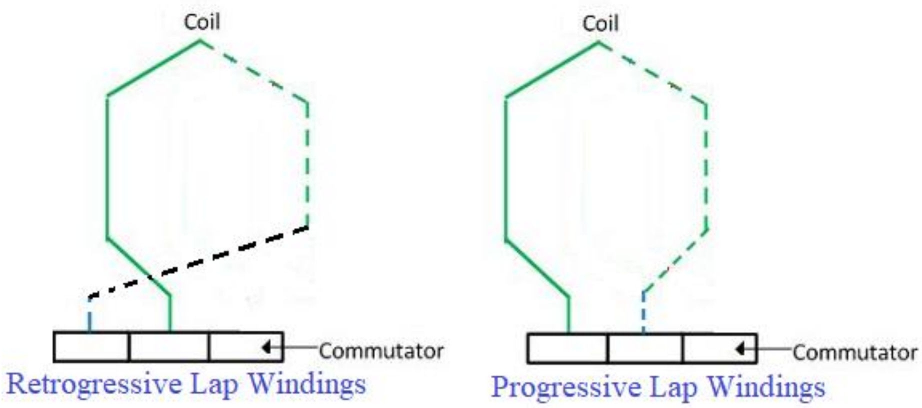


Fig. 25, Progressive and Retrogressive winding

c) Lap winding
d) Wave winding

Let us define some important terms before starting the windings:

Assuming a machine with number of coils (C), then the total number of coil sides is 2C.

Winding pitch (Y): it is the distance between the beginnings of two consecutive turns.

$$Y = Y_B - Y_F \qquad (for\ lap\ winding)$$

$$Y = Y_B + Y_F \qquad (for\ wave\ winding)$$

Back pitch (Y_B): is the distance between the two coil sides (S and F) of one coil, and it must be odd number and can be given by:

$$Y_B = \frac{2C}{2P} + m$$

Front pitch (Y_F): is the distance between the F coil sides of one coil and the S coil side of next coil that are connected together to the same commutator segment, and it must be odd number and can be given by:

$$Y_F = \frac{2C}{2P} - m$$

Where *m* is an arbitrary integer number define the -plex and equal:

+1, for progressive simplex winding & -1, for retrogressive simplex winding

+2, for progressive duplex winding & -2, for retrogressive duplex winding

+3, for progressive triplex winding & -3, for retrogressive triplex winding

+4, for progressive quadruplex winding & -4, for retrogressive quadruplex winding

Commutator Pitch (Y_C): It is the distance (measured in commutator segments) between the segments to which the (S and F) of a coil are connected.

For simplex lap winding,

$Y_c = +1$ (Progressive Lap)

$Y_c = -1$ (Retrogressive Lap). Generally, $\pm m$

Y_B, Y_F, Y and Y_c are illustrated in Fig. 26 (a) for lap winding (b) for wave winding.

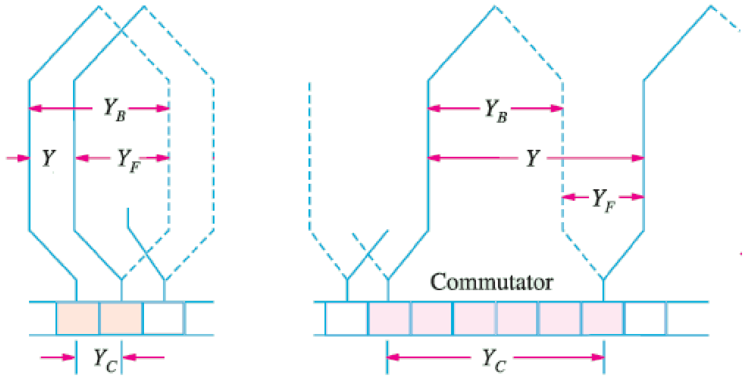
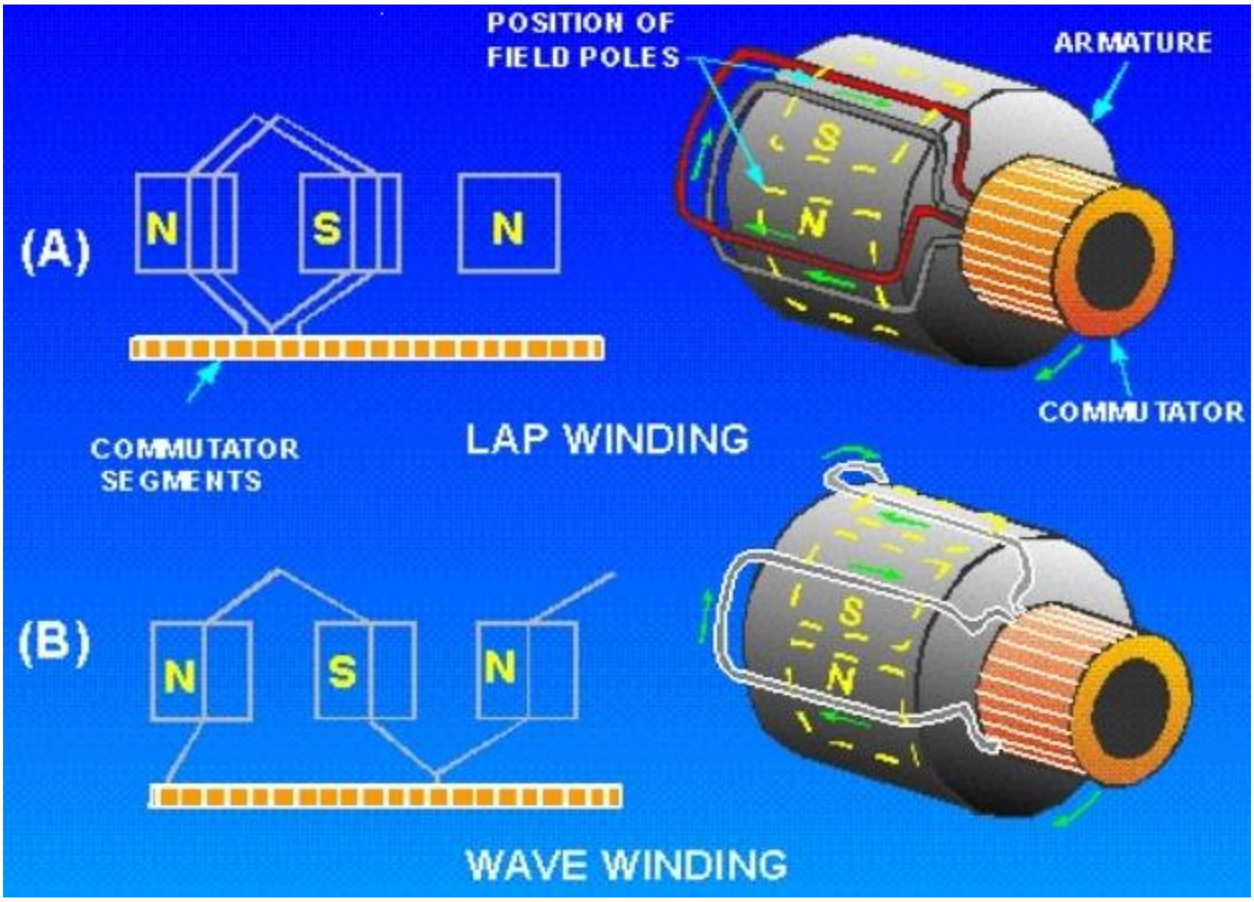


Fig. 26, (a) Lap winding (b) Wave winding



Progressive Simplex Lap Winding (General Rules)

- Number of slots (S) = Number of Coils (C) = Number of Commutator segments
- $Y_B > Y_F$ and both are odd number
- Y is even number
- $Y_c = +1$

In DC winding diagram, we numbered the coil sides consecutively, i.e., 1, 2, 3, … etc. such that odd numbers are assigned to top coil sides that represented by solid lines, and the even numbers are assigned to lower coil sides that represented by dashed lines.

Example

Design a 4-pole, simplex lap-winding suitable for an armature containing 20 slots.

$Y_B = (40/4)+1=11$, $Y_F = (40/4)-1 = 9$, $Y = 11 - 9 = 2$, $Y_C = +1$

Example

Draw a developed diagram of a 2-layer simplex lap-winding for a 4-pole generator with 16 slots.

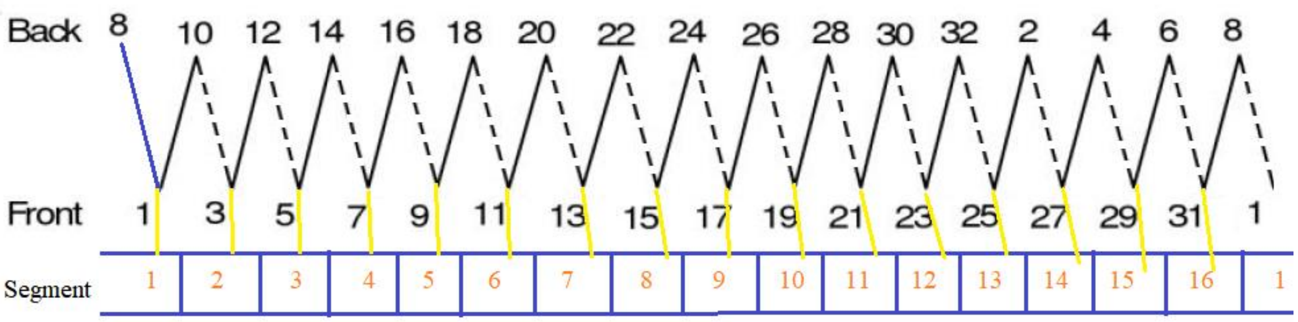
For double layer windings, the number of coils (C) =16,

The number of commutator segments = 16

Number of coil sides =2C= 32; pole pitch = 2C/2P = 32/4=8 coil side per pole;

Y_B = 8+1 = 9, Y_F = 8-1 = 7. The winding table is given as:

Segment	Back Connection (Y_B)	Front Connection (Y_F)
#1	From 1 to (1+9)=10	From 10 to (10-7)=3
#2	From 3 to (3+9)=12	From 12 to (12-7)=5
#3	From 5 to (5+9)=14	From 14 to (14-7)=7
#4	From 7 to (7+9)=16	From 16 to (16-7)=9
#5	From 9 to (9+9)=18	From 18 to (18-7)=11
#6	From 11 to (11+9)=20	From 20 to (20-7)=13
#7	From 13 to (13+9)=22	From 22 to (22-7)=15
#8	From 15 to (15+9)=24	From 24 to (24-7)=17
#9	From 17 to (17+9)=26	From 26 to (26-7)=19
#10	From 19 to (19+9)=28	From 28 to (28-7)=21
#11	From 21 to (21+9)=30	From 30 to (30-7)=23
#12	From 23 to (23+9)=32	From 32 to (32-7)=25
#13	From 25 to (25+9)=34-32=2	From 2 to (2-7)=-5+32=27
#14	From 27 to (27+9)=36-32=4	From 4 to (4-7)=-3+32=29
#15	From 29 to (29+9)=38-32=6	From 6 to (6-7)=-1+32=31
#16	From 31 to (31+9)=40-32=8	From 8 to (8-7)=1



The last coil side (8) joins to the first coil side (1), so that the winding closes upon itself. Developed winding diagram, shown in Fig. 27, is obtained by imagining the cylindrical surface of the armature is cut and flattened out.

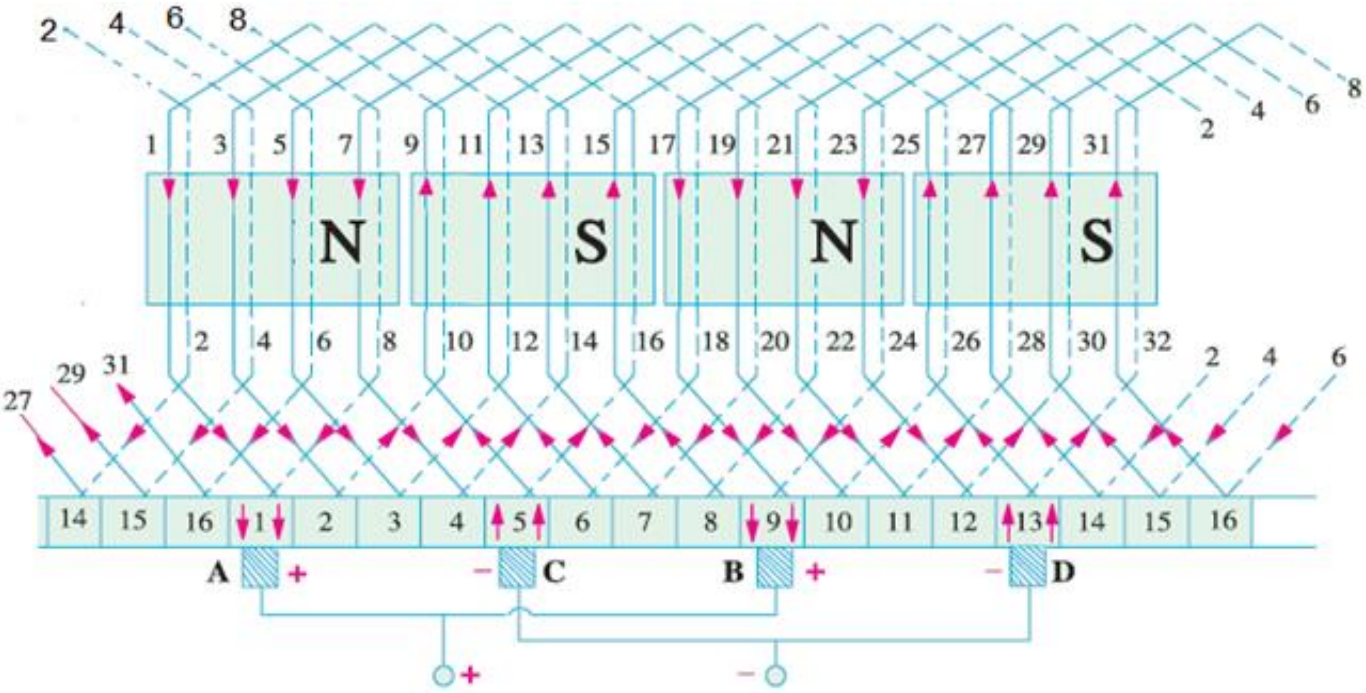
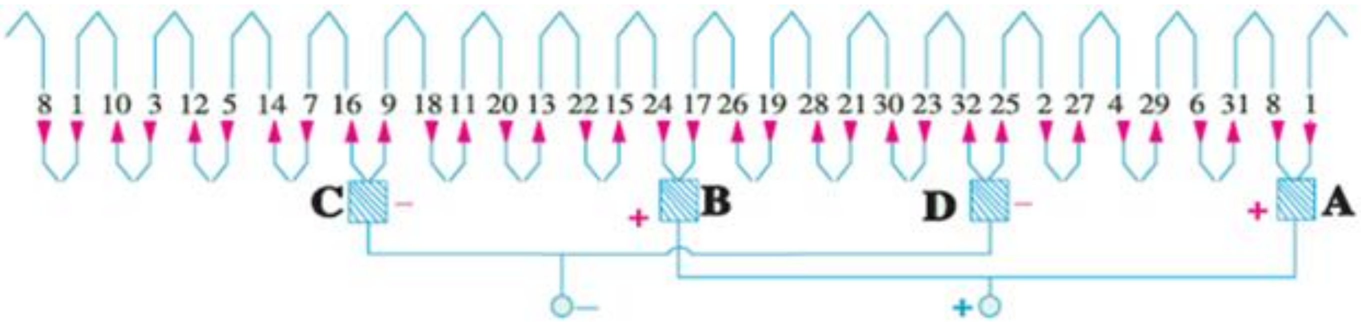
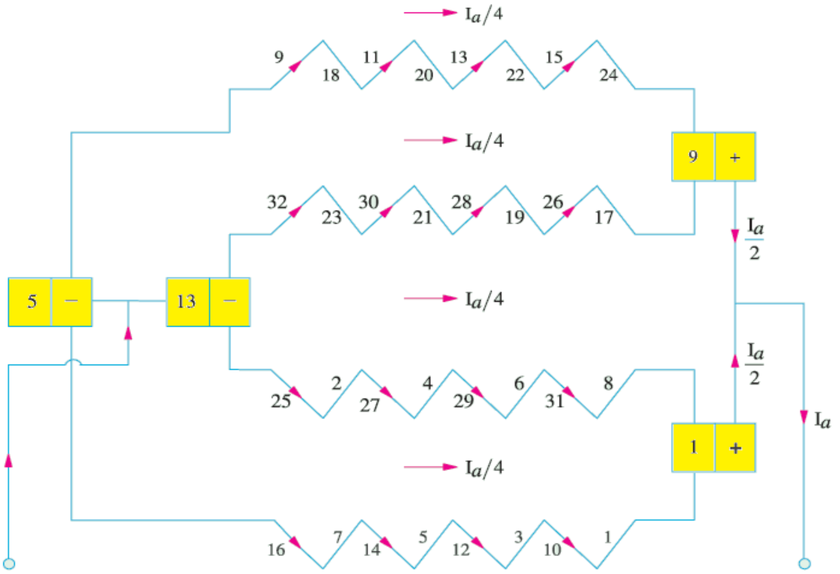
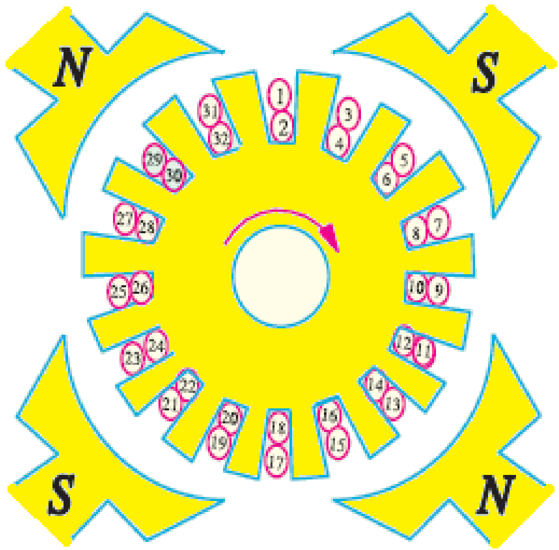


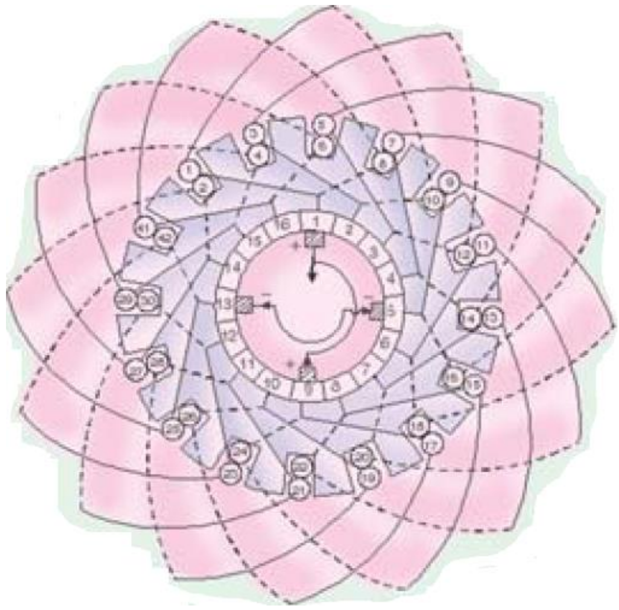
Fig. 27, Developed Lap winding



Equivalent Ring winding







<u>Example:</u>

4-pole, simplex progressive lap wound armature contains 21 slots and two coil sides per slot. Calculate:

a) Back and Front Pitch (Y_B & Y_F)

b) Winding Pitch (Y)

c) Commutator Pitch (Y_C)

The pole pitch = S/2P = 21/4 = 5.25 (slots)

Ignoring the fraction 0.25, then the pole pitch ≈ 5 (slots) ≈ 10 (coil sides)

Y_B = 10 +1 = 11, Y_F = 10 – 1 = 9, Y = 11 – 9 = 2, Y_C = +1

<u>Summarizing the simplex lap windings, we have</u>

1. The total number of brushes is equal to the number of poles.

2. There are as many parallel paths in the armature as the number of poles. That is why such a winding is sometimes known as 'multiple circuit' or 'parallel' winding.

In general, number of parallel paths in armature = $m\times 2P$ where m is the (plex) of the lap winding. For example, a 6-pole duplex lap winding has (6 × 2) = 12 parallel paths in its armature.

3. Obviously, if I_a is the total current supplied by the generator, then current carried by each parallel path is $I_a/(m\times 2P)$. In case of simplex (m=1), the current in each parallel path is $I_a/4$.

4. The e.m.f. between the +ve and −ve brushes is equal to the e.m.f. generated in any one of the parallel paths. If *Z* is the total number of armature coil sides and 2*P* the number of poles, then the number of armature conductors (connected in series) in any parallel path is $Z/(m\times 2P)$.

Progressive Simplex Wave Windings:

The main difference between Lap and Wave windings is the commutator connection.

Winding pitch (Y): it is the distance (measured in coil sides) between the beginnings of two consecutive turns.

$$Y = \frac{2C \ \pm 2}{P} = Y_B + Y_F$$

Average Pitch (Y_A):

$$Y_A = \frac{Y_B + Y_F}{2} = \frac{2C \ \pm 2}{2P}$$

Commutator Pitch (Y_C): It is the distance (measured in commutator segments) between the segments to which the (S and F) of a coil are connected.

$$Y_C = \frac{Number\ of\ Commutator\ Sigments \pm 1}{P}$$

OR $\quad Y_C = Y_A$

The following points must be kept in mind:

1- Y must be a whole even number
2- Both Y_B and Y_F are odd
3- Y_B and Y_F may be equal OR differ by 2. In case of equal, $Y_B = Y_F = Y_A$. In case of differ by 2, then $Y_B = Y_A+1$ and $Y_F = Y_A - 1$

Example:

Design a 4-pole simplex wave winding for an armature with 21 slots.

2C = 42 coil sides

$$Y_A = \frac{2C \ \pm 2}{2P} = \frac{42 \pm 2}{4} = 10\ or\ 11$$

First Choice: we select $Y_A = 10$;

$Y_B = 10 + 1 = 11$, (odd)

$Y_F = 10 - 1 = 9$, (odd)

$Y = Y_B + Y_F = 11 + 9 = 20$, (even)

$Y_C = Y_A = 10$

Second Choice: we select $Y_A = 11$;

$Y_B = 11$ (odd) $Y_F = 11$ (odd)

$Y = 22$ (even) $Y_C = Y_A = 11$

The winding table (for the second choice) is given as:

Segment	Back Connection (Y_B)	Front Connection (Y_F)
#18	From 1 to (1+11)=12	From 12 to (12+11)=23
#8	From 23 to (23+11)=34	From 34 to (34+11-42)=3
#19	From 3 to (3+11)=14	From 14 to (14+11)=25
#9	From 25 to (25+11)=36	From 36 to (36+11-42)=5
#20	From 5 to (5+11)=16	From 16 to (16+11)=27
#10	From 27 to (27+11)=38	From 38 to (38+11-42)=7
#21	From 7 to (7+11)=18	From 18 to (18+11)=29
#11	From 29 to (29+11)=40	From 40 to (40+11-42)=9
#1	From 9 to (9+11)=20	From 20 to (20+11)=31
#12	From 31 to (31+11)=42	From 42 to (42+11-42)=11
#2	From 11 to (11+11)=22	From 22 to (22+11)=33
#13	From 33 to (33+11-42)=2	From 2 to (2+11)=13
#3	From 13 to (13+11)=24	From 24 to (24+11)=35
#14	From 35 to (35+11-42)=4	From 4 to (4+11)=15
#4	From 15 to (15+11)=26	From 26 to (26+11)=37
#15	From 37 to (37+11-42)=6	From 6 to (6+11)=17
#5	From 17 to (17+11)=28	From 28 to (28+11)=39
#16	From 39 to (39+11-42)=8	From 8 to (8+11)=19
#6	From 19 to (19+11)=30	From 30 to (30+11)=41
#17	From 41 to (41+11-42)=10	From 10 to (10+11)=21
#7	From 21 to (21+11)=32	From (32+11-42)=1

Since we come back to coil side number 1, the winding is closed.

The first value of segment number is chosen 18 and we must satisfy Y_C=11

The developed winding diagram is shown in Fig. 28.

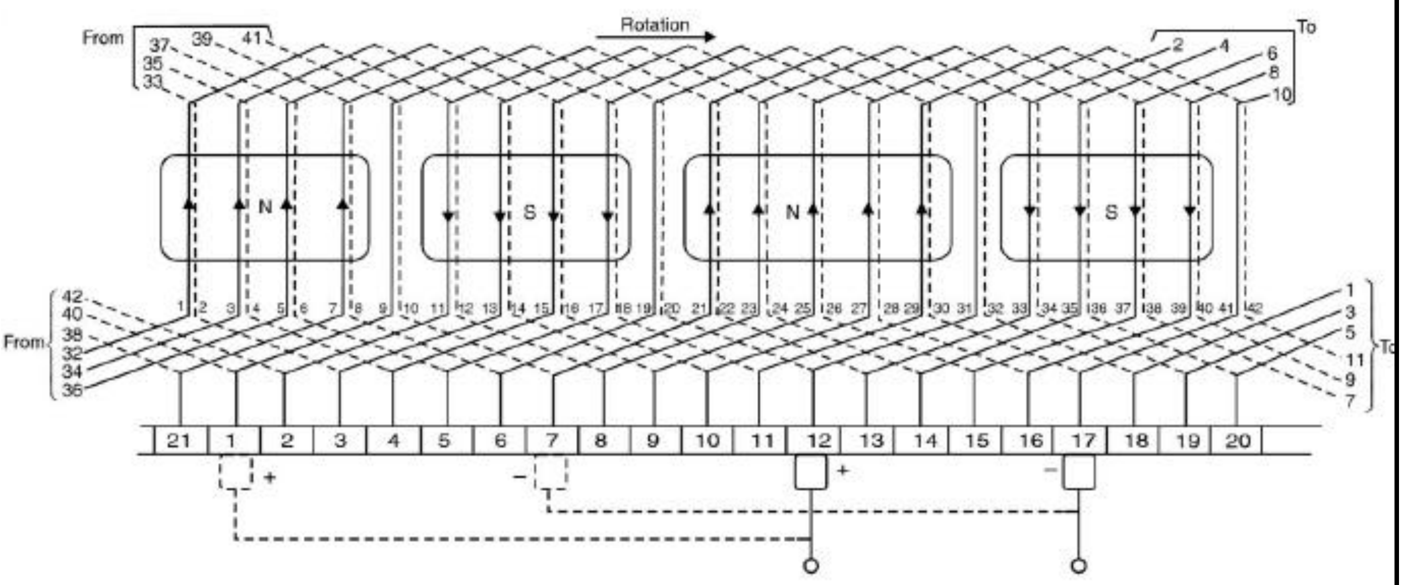
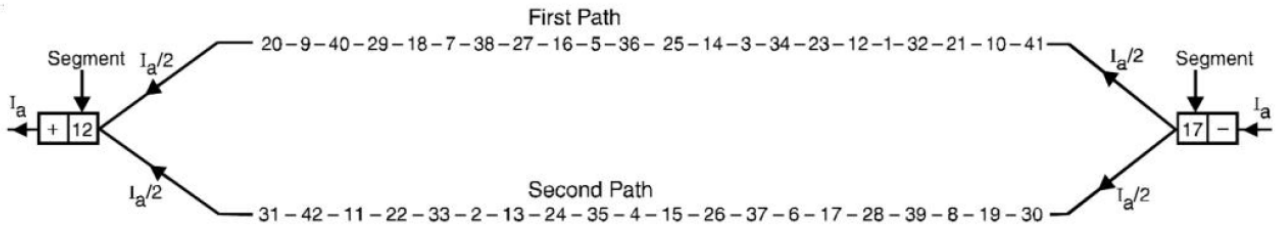


Fig. 28, Developed wave winding



At this point:

- Only two brushes are used
- The armature winding is divided into only two parallel paths, each path has half of the coil sides. And the current in each path is half the main current.
- The wave winding is possible only if $Y_A = Y_C$ is whole number

Types of Generators

A DC machine has two distinct circuits, a field circuit and an armature circuit. The mmfs produced by these two circuits are at quadrature—the field mmf is along the direct axis and the armature mmf is along the quadrature axis. A simple schematic representation of the DC machine is shown in Fig. 29.

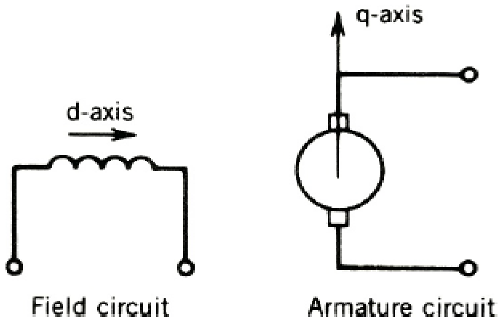


Fig. 29, Representation of DC machine

The field circuit and the armature circuit can be interconnected in various ways to provide a wide variety of performance characteristics. Also, the field poles can be excited by two field windings; a shunt field winding and a series field winding. The shunt winding has a large number of turns and takes only a small current (less than 5% of the rated armature current). This winding can be connected across the armature (i.e., parallel with it), hence the name shunt winding.

The series winding has fewer turns but carries a large current. It is connected in series with the armature, hence the name series winding.

If both shunt and series windings are present, the series winding is wound on top of the shunt winding.

A rheostat is normally included in the circuit of the field winding to control the field current and thereby to vary the field mmf.

Generators are usually classified according to the way in which their fields are excited. Generators may be divided into:

(*a*) separately-excited generators (*b*) self-excited generators.

(*a*) Separately-excited generators are those whose field magnets are energised from an independent external source of DC current. Fig. 30.

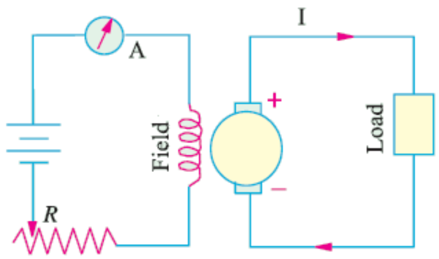


Fig. 30, Separately Excited DC machine

Field excitation may also be provided by permanent magnets. This may be considered as a form of separately excited machine, the permanent magnet providing the separate but constant excitation.

(*b*) **Self-excited generators** are those whose field magnets are energised by the current produced by the generators themselves.

There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature.

(*i*) Shunt wound

The field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them (Fig. 31).

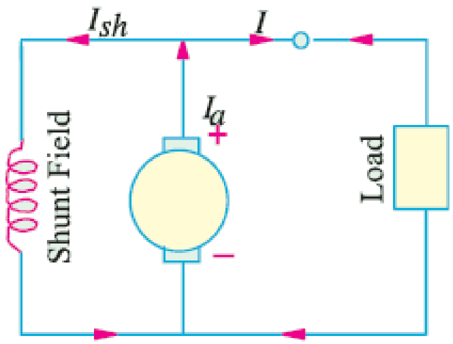


Fig. 31, Shunt DC machines

(*ii*) Series Wound

the field windings are joined in series with the armature conductors (Fig. 32).

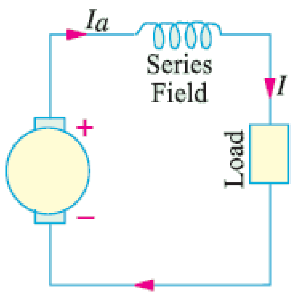


Fig. 32, Series DC machines

(*iii*) Compound Wound

It is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt as shown in Fig. 33.

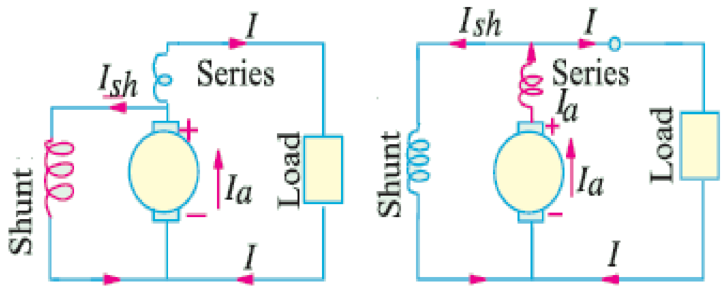
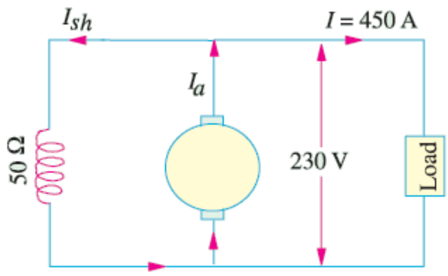


Fig. 33, a) Short compound b) Long compound

Example:

A shunt generator delivers 450 A at 230 V and the resistance of the shunt field and armature are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f.



Current through shunt field winding is

$$I_{sh} = 230/50 = 4.6 \text{ A}$$

Load current $I = 450$ A

$\therefore$ Armature current $I_a = I + I_{sh}$

$$= 450 + 4.6 = 454.6 \text{ A}$$

Armature voltage drop

$$I_a R_a = 454.6 \times 0.03 = 13.6 \text{ V}$$

Now E_g = terminal voltage + armature drop

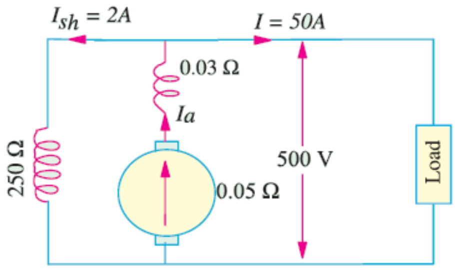
$$= V + I_a R_a$$

$\therefore$ e.m.f. generated in the armature

$$E_g = 230 + 13.6 = 243.6 \text{ V}$$

Example:

A long-shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop.



$$I_{sh} = 500/250 = 2 \text{ A}$$

Current through armature and series winding is

$$= 50 + 2 = 52 \text{ A}$$

Voltage drop on series field winding

$$= 52 \times 0.03 = 1.56 \text{ V}$$

Armature voltage drop

$$I_aR_a = 52 \times 0.05 = 2.6 \text{ V}$$

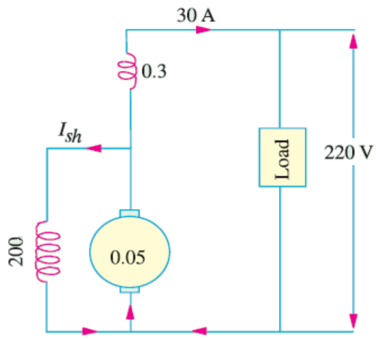
Drop at brushes = 2 × 1 = 2 V

Now, $E_g = V + I_aR_a + \text{series drop} + \text{brush drop}$

$$= 500 + 2.6 + 1.56 + 2 = \mathbf{506.16 \text{ V}}$$

Example:

A short-shunt compound generator delivers a load current of 30 A at 220 V, and has armature, series-field and shunt-field resistances of 0.05 Ω, 0.3 Ω and 200 Ω respectively. Calculate the induced e.m.f. and the armature current. Allow 1.0 V per brush for contact drop.



Voltage drop in series winding = 30 × 0.3 = 9 V

Voltage across shunt winding = 220 + 9 = 229 V

$$I_{sh} = 229/200 = 1.145 \text{ A}$$

$$I_a = 30 + 1.145 = 31.145 \text{ A}$$

$$I_aR_a = 31.145 \times 0.05 = 1.56 \text{ V}$$

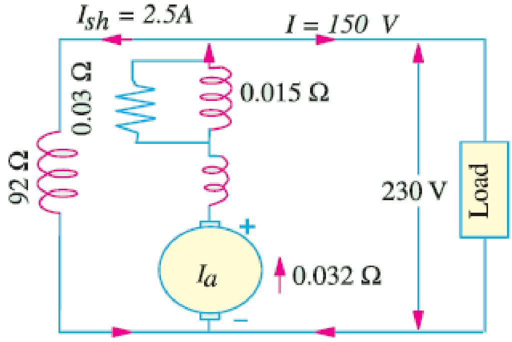
$$\text{Brush drop} = 2 \times 1 = 2 \text{ V}$$

$$E_g = V + \text{series drop} + \text{brush drop} + I_aR_a$$

$$= 220 + 9 + 2 + 1.56 = 232.56 \text{ V}$$

Example:

In a long-shunt compound generator, the terminal voltage is 230 V when generator delivers 150 A. Determine **(i)** induced e.m.f. **(ii)** total power generated and **(iii)** distribution of this power. Given that shunt field, series field, divertor and armature resistance are 92 Ω, 0.015 Ω, 0.03 Ω and 0.032 Ω respectively.



$$I_{sh} = 230/92 = 2.5 \text{ A}$$
$$I_a = 150 + 2.5 = 152.5 \text{ A}$$

Since series field resistance and divertor resistances are in parallel their combined resistance is

$$= 0.03 \times 0.015/0.045 = 0.01\ \Omega$$

Total armature circuit resistance is

$$= 0.032 + 0.01 = 0.042\ \Omega$$

Voltage drop $= 152.5 \times 0.042 = 6.4$ V

(i) Voltage generated by armature

$$E_g = 230 + 6.4 = \mathbf{236.4\ V}$$

(ii) Total power generated in armature

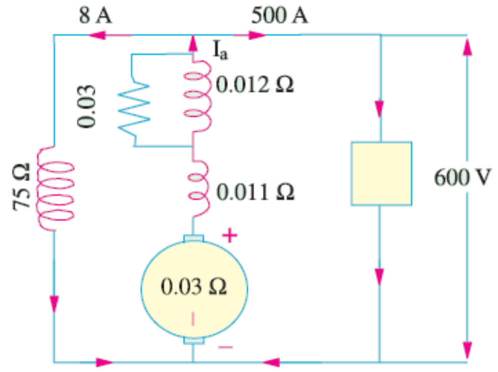
$$E_g I_a = 236.4 \times 152.5 = \mathbf{36{,}051\ W}$$

(iii) Power lost in armature

$I_a R_a$	=	$152.5^2 \times 0.032$	=	744 W
Power lost in series field and divertor	=	$152.5^2 \times 0.01$	=	232 W
Power dissipated in shunt winding	=	$VI_{sh} = 230 \times 0.01$	=	575 W
Power delivered to load	=	230×150	=	34500 W
		Total	=	36,051 W.

Example:

The following information is given for 300-kW, 600-V, long-shunt compound generator Shunt field resistance = 75 Ω, Armature resistance including brush resistance = 0.03 Ω, commutating field winding resistance = 0.011 Ω, series field resistance = 0.012 Ω, divertor resistance = 0.036 Ω. When the machine is delivering full load, calculate the voltage and power generated by the armature.



$$\text{Power output} = 300{,}000 \text{ W}$$
$$\text{Output current} = 300{,}000/600$$
$$= 500 \text{ A}$$
$$I_{sh} = 600/75 = 8 \text{ A},$$
$$I_a = 500 + 8 = 508 \text{ A}$$

Since the series field resistance and divertor resistance are in parallel

$$= \frac{0.012 \times 0.036}{0.048} = 0.009\ \Omega$$

Total armature circuit resistance

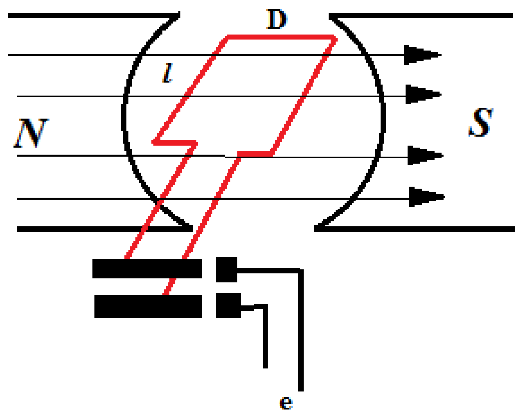
$$= 0.03 - 0.011 + 0.009 = 0.05\ \Omega$$
$$\text{Voltage drop} = 508 \times 0.05 = 25.4 \text{ V}$$

Voltage generated by armature

$$= 600 + 25.4 = 625.4 \text{ V}$$
$$\text{Power generated} = 625.4 \times 508 = 317{,}700$$
$$W = \mathbf{317.7\ kW}$$

Generated EMF of DC Generator:

Consider the elementary DC generator shown in the following figure:



D is the diameter of the circular path (rotor),

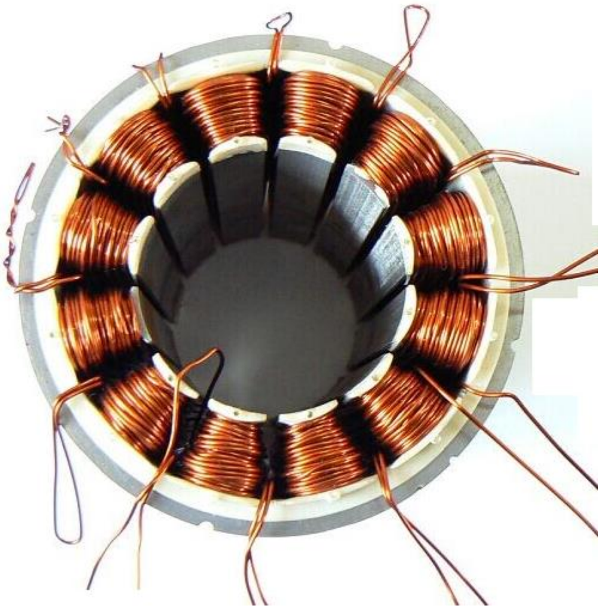
l is the effective conductor length in meter.

The induced emf in a conductor moving in a magnetic field is given by:

$$e = B\, l\, v$$

where, B is the flux density per pole (Wb/m^2)

v is the velocity (m/s)



the velocity and flux density are given as:

$$v = \pi\ D\ \frac{N}{60} \qquad \&\ B = \frac{\varphi \times 2P}{l \times \pi D}$$

where N is the rotational speed in rpm

substituting the values of v and B in the emf equation,

$$e = \frac{\varphi \times 2P}{l \times \pi D}\ l\ \pi\ D\ \frac{N}{60} = \frac{\varphi \times 2P \times N}{60}$$

To get an expression for all conductors connected in series in the machine

$$E_a = \frac{\varphi \times 2P \times N}{60} \times \frac{Z}{a}$$

Where Z is total number of conductors,

a is the number of parallel paths (a = m×2P in case of lap winding & a =2 in case of wave winding)

Multiplying both num and denum by 2π

$$E_a = \frac{\varphi \times 2P \times 2\pi N}{2\pi \times 60} \times \frac{Z}{a}$$

$$E_a = \frac{2PZ}{2\pi a} \times \varphi \times \omega$$

$$E_a = K_a \times \varphi \times \omega$$

Where K_a is the machine constant and is given by

$$K_a = \frac{2PZ}{2\pi a}$$

By the same way, the developed torque from the DC generator can be obtained by

$$T = K_a \times \varphi \times I_a$$

Example:

A four-pole DC machine has an armature of radius 12.5 cm and an effective length of 25 cm. The poles cover 75% of the armature periphery. The armature winding consists of 33 coils, each coil having seven turns. The coils are accommodated in 33 slots. The average flux density under each pole is 0.75T.

1. If the armature is simplex lap-wound,

(a) Determine the armature constant K_a.

(b) Determine the induced armature voltage when the armature rotates at 1000 rpm.

(c) Determine the current in the coil and the electromagnetic torque developed when the armature current is 400 A.

(d) Determine the power developed by the armature.

2. If the armature is wave-wound, repeat parts (a) to (d) above. The current rating of the coils remains the same as in the lap-wound armature.

In case of simplex lap winding, a = 2P = 4

Z = 33 coils × 7 turns × 2 = 462

$$K_a = \frac{2PZ}{2\pi a} = \frac{4 \times 462}{2\pi \times 4} = 73.5296$$

Area under the pole = (2π×0.125×0.25×0.75) / 4 = 36.8 mm^2

Φ = B×area under the pole = 0.75×36.8×10^{-3} = 0.0276 Wb

ω = 2π×1000/60 = 104.72 rad/s

E_a = 73.5296 × 0.0276 × 104.72 = 212.5205 V

T = 73.53 × 0.0276 × 400 = 811.7668 Nm

I_{coil} = 400/ a = 400/4 = 100 A

Power developed by the armature = E_a I_a = 212.5205 × 400 = 85008.2 W

2. in case of wave winding: a = 2

You can continue solving the problem as before, taking in consideration that the armature current I_a will be 2×100 = 200 A

$$K_a = \frac{2PZ}{2\pi a} = \frac{4 \times 462}{2\pi \times 2} = 147.0592$$

E_a = 147.0592 × 0.0276 × 104.72 = 425.041 V

T = 147.0592 × 0.0276 × 200 = 811.7668 Nm

Power developed by the armature = E_a I_a = 425.041 × 200 = 85008.2 W

Example:

An 8-pole DC generator has 500 armature conductors, and a useful flux of 0.05 Wb per pole. What will be the e.m.f. generated if it is lap-connected and runs at 1200 rpm? What must be the speed at which it is to be driven to produce the same e.m.f. if it is wave-wound?

In case of lap winding, a = 8

$$K_a = \frac{2PZ}{2\pi a} = \frac{Z}{2\pi} = \frac{500}{2\pi}$$

$$E_a = K_a \times \varphi \frac{2\pi \times N}{60} = \frac{500}{2\pi} \times 0.05 \times \frac{2\pi \times 1200}{60} = 500\,V$$

In case of wave, a = 2

$$K_a = \frac{2PZ}{2\pi a} = \frac{8 \times 500}{4\pi} = \frac{2000}{2\pi}$$

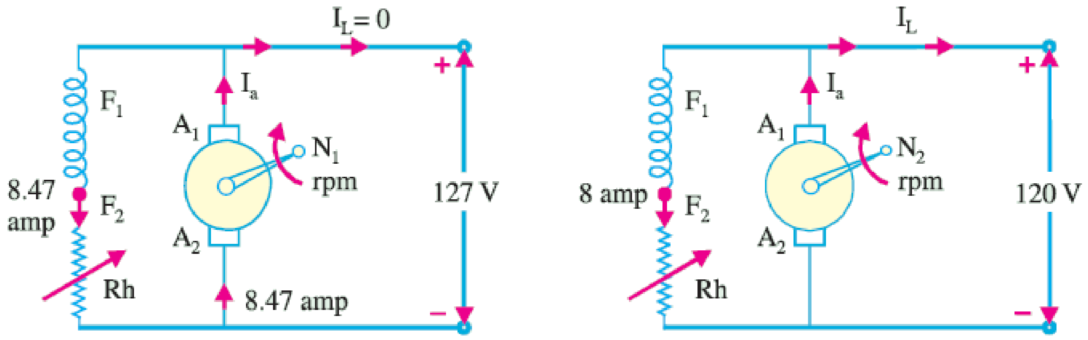
$$500 = \frac{2000}{2\pi} \times 0.05 \times \frac{2\pi \times N}{60}$$

N = 300 rpm

Example:

A DC shunt generator has an induced voltage on open-circuit of 127 volts. When the machine is on load, the terminal voltage is 120 volts. Find the load current if the field circuit resistance is 15 ohms and the armature-resistance is 0.02 ohm.

If the generator speed is N_1 and equal 1000 rpm at no load, what will be the speed N_2 at this loading condition?



(*a*) Generator on no-load (*b*) Loaded Generator

At no load, $I_f = 127/15 = 8.4667$ A

Also, $E_a = V_t + I_a R_a = 127 + 8.4667 \times 0.02 = 127.1693$ V and it will be constant

After loading V_t becomes 120 V

$E_a = V_t + I_a R_a \quad \rightarrow \quad 127.1693 = 120 + 0.02 I_a \quad \rightarrow \quad I_a = 358.4667$ A

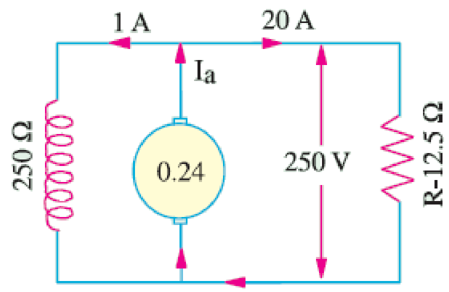
Load current $I_L = 358.4667 + 8 = 366.4667$ A

The emf at no load = emf at load

$K_a I_f N_1 = K_a I_f N_2 \quad \rightarrow 8.4667 \times 1000 = 8 \times N_2 \quad \rightarrow \quad N_2 = 1058.3375$ rpm

<u>Example</u>

An 8-pole DC shunt generator with 778 wave-connected armature conductors and running at 500 r.p.m. supplies a load of 12.5 Ω resistance at terminal voltage of 250 V. The armature resistance is 0.24 Ω and the field resistance is 250 Ω. Find the armature current, the induced e.m.f. and the flux per pole.



Field current $I_f = 250/250 = 1.0$ A

Load current $I_L = 250/12.5 = 20$ A

Then the armature current $I_a = 20+1 = 21$ A

$E_a = 250 + 21 \times 0.24 = 255.04$ A

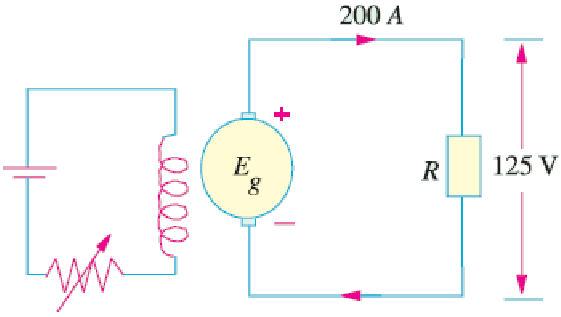
$\Omega = 2\pi \times 500/60 = 52.3599$ rad/s

$$255.04 = \frac{8 \times 778}{2\pi \times 2} \times \emptyset \times 52.3599$$

Φ = 9.8344 mWb

Example:

A separately excited DC generator, when running at 1000 r.p.m. supplied 200 A at 125 V. What will be the load current when the speed drops to 800 r.p.m. if I_f is unchanged? Given that the armature resistance = 0.04 ohm and brush drop = 2 V.



The load resistance R_L=125/200 = 0.625 Ω

E_g = 125 + 200×0.04 + 2 = 135 V this at N_1 =1000 rpm, and field current I_f

$$135 = K_a I_f \frac{2\pi \times 1000}{60}$$

$K_a\ I_f$ = 1.2892

Now, at N_2 = 800 rpm

$$E_g = K_a I_f \frac{2\pi \times 800}{60} = 1.2892 \frac{2\pi \times 800}{60} = 108\ V$$

$E_g = I_a(R_a+R_L)+2$

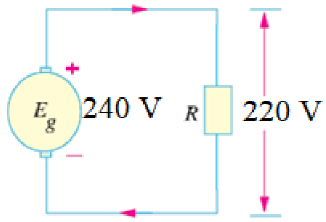
108 = I_a (0.04 + 0.625) + 2 → I_a = 159.3985 A

Example:

A 4-pole, wave-wound with 8 turns per coil, DC machine runs at 900 rpm and has a terminal and induced voltages of 220 V and 240 V respectively. The armature circuit resistance is 0.2 Ω. Calculate the armature current and the number of armature coils if the air-gap flux/pole is 10 mWb.

$$E_g = V_t + I_a R_a$$

$$I_a = \frac{240 - 220}{0.2} = 100\ A$$



$$E_a = \frac{2PZ}{2\pi a} \times \varphi \frac{2\pi \times N}{60}$$

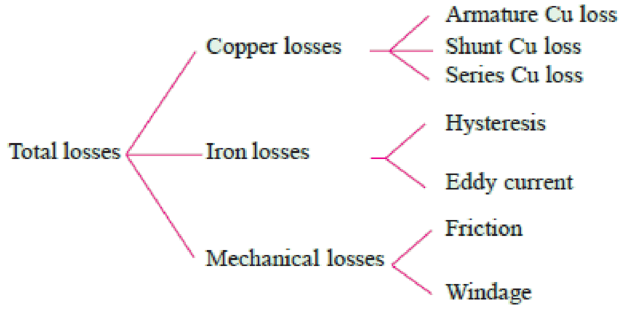
$$240 = \frac{4 \times Z}{2\pi \times 2} \times 10 \times 10^{-3} \times \frac{2\pi \times 900}{60}$$

240 = 0.3 Z → Z = 800 conductors

Total number of turns = 800/2 = 400 turns

Total number of coils = 400/8 = 50 coil

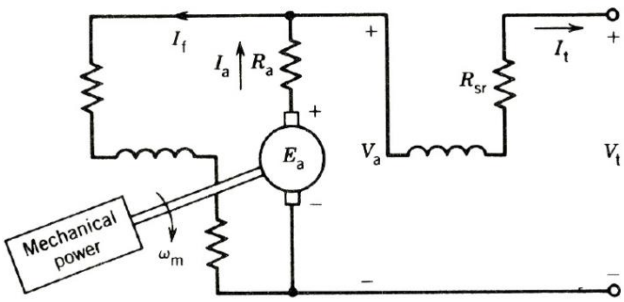
Efficiency of DC generators:

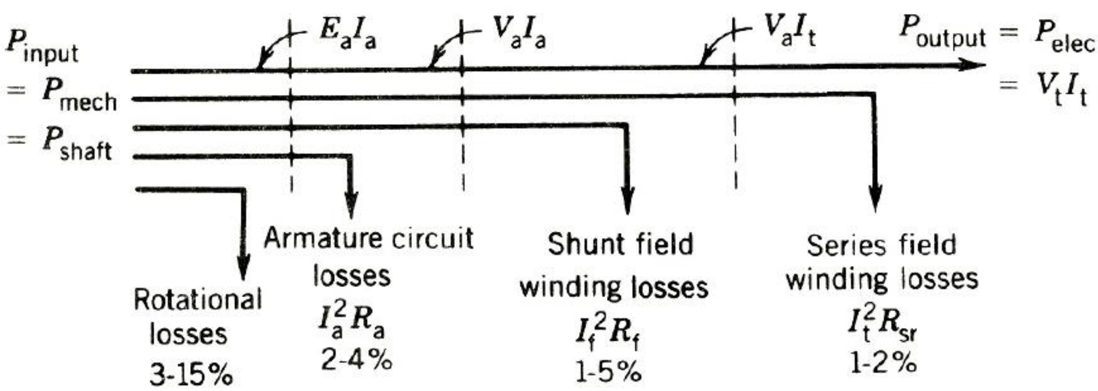


The output power from DC generator $P_{out} = V_t I_t$

The efficiency can be calculated by

$$\eta = \frac{P_{out}}{P_{out} + Losses}$$





There are 3 different types of efficiencies:

Mechanical Efficiency:

$$\eta_m = \frac{Total\ Power\ generated\ in\ armature}{Total\ Mechanical\ power\ input} = \frac{E_a\ I_a}{T\ \omega}$$

Electrical Efficiency:

$$\eta_e = \frac{Electrical\ Power\ Output\ to\ the\ load}{Total\ Power\ generated\ in\ armature} = \frac{V_t\ I_t}{E_a\ I_a}$$

Overall Efficiency:

$$\eta_t = \frac{Electrical\ Power\ Output\ to\ the\ load}{Total\ Mechanical\ power\ input} = \frac{V_t\ I_t}{T\ \omega}$$

Constant and Variable losses

The losses in a d.c. generator (or d.c. motor) may be sub-divided into (*i*) constant losses (*ii*) variable losses.

Total losses = Constant losses + Variable losses

(*i*) **Constant losses.** Those losses in a d.c. generator which remain constant at all loads are known as *constant losses*. The constant losses in a d.c. generator are :

(*a*) iron losses,
(*b*) mechanical losses,
(*c*) shunt field losses **Note.** Field Cu loss is constant for shunt and compound generators.

(*ii*) **Variable losses.** Those losses in a d.c. generator which vary with load are called *variable losses*. The variable losses in a d.c. generator are :

(*a*) Copper loss in armature winding ($I_a^2 R_a$)
(*b*) Copper loss in series field winding ($I_{se}^2 R_{se}$)

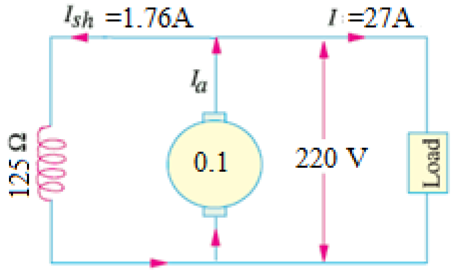
Condition for maximum Efficiency

As before, the generator efficiency is maximum when the variable losses equal the constant losses.

Example:

The armature winding of a 4-pole, 220 V DC shunt generator is lap connected. There are 120 slots, each slot containing two coil sides, each coil side containing 5 conductors. The flux per pole is 25 mWb and current delivered by the generator is 27 A. The resistance of armature and field circuit are 0.1 and 125 Ω respectively. If the rotational losses amount to be 780 W and the drop voltage on each brush to be 1.5 V find,

(i) Prime Mover Speed in rpm,
(ii) Electromagnetic torque,
(iii) Input Power in HP,
(iv) Efficiency.



I_{sh} = 220 / 125 = 1.76 A

I_a = 27 + 1.76 = 28.76 A

E_a = 28.76×0.1 + 1.5×2 + 220 = 225.876 V

Z = 120×5×2 = 1200 conductors

Since lap winding, *a* = 2P = 4

K_a = 1200 / 2π = 190.9859

Since $E_a = K_a \times \Phi \times \omega$

ω = 225.876 / (190.9859 × 0.025) = 47.3074 rad/s.

N = 60×ω / (2π) = 60 × 47.3074 / (2π) = 451.752 rpm ##

Electromagnetic Power (P_{em}) = E_a I_a = 225.876 × 28.76 = 6496.1938 W

$P_{em} = T_{em} \times \omega$

T_{em} = P_{em} / ω = 6496.1938 / 47.3074 = 137.3189 N.m ####

Input power (P_{in}) = P_{em} + $P_{rotational}$ = 6496.1938 + 780 = 7276.1938 W

I HP = 746 W

P_{in} = 7276.1938 / 746 = 9.7536 HP ####

P_{out} = 220 × 27 = 5940 W

Efficiency = P_{out} / P_{in} = (5940 / 7276.1938) ×100 % = 81.6361%

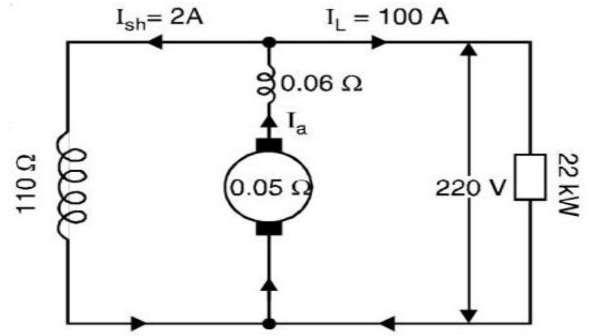
Example:

A long-shunt compound generator running at 1000 rpm supplies 22 kW at a terminal voltage of 220 V. The resistances of the armature, shunt field and series field are 0.05Ω, 110Ω and 0.06Ω respectively. The overall efficiency of the above load is 88%. Find:

i. Cu losses

ii. Iron and friction losses

iii. Torque exerted by the prime mover.



$$I_L = \frac{22 \times 10^3}{220} = 100\ \text{A}; \qquad I_{sh} = \frac{220}{110} = 2\text{A}$$

$$I_a = I_L + I_{sh} = 100 + 2 = 102\ \text{A}$$

Voltage drop in series field winding $= I_a R_{se} = 102 \times 0{\cdot}06 = 6{\cdot}12$ V

(*i*) Armature Cu loss $= I_a^2 R_a = (102)^2 \times 0{\cdot}05 = 520{\cdot}2$ W

Series field Cu loss $= I_a^2 R_{se} = (102)^2 \times 0{\cdot}06 = 624{\cdot}3$ W

Shunt field Cu loss $= V I_{sh} = 220 \times 2 = 440$ W

Total Cu losses $= 520{\cdot}2 + 624{\cdot}3 + 440 =$ **1584·5 W**

(*ii*) Output $= 22$ kW $= 22000$ W ; Input $= 22000/0{\cdot}88 = 25000$ W

Total losses = Input − Output $= 25000 - 22000 = 3000$ W

∴ Iron and friction losses = Total losses − Total Cu losses

$= 3000 - 1584{\cdot}5 =$ **1415·5 W**

(*iii*) Let T N-m be the torque exerted by the prime mover.

Input power $= T \times$ Angular velocity

or $$25000 = T \times \frac{2\pi N}{60}$$

∴ $$T = \frac{25000 \times 60}{2\pi \times 1000} = \mathbf{238{\cdot}74\ N\text{-}m}$$

Example:

A shunt generator delivers 195 A at terminal voltage of 250 V. The armature resistance and shunt field resistance are 0.02 Ω and 50 Ω respectively. The iron and friction losses equal 950 W. Find

(a) E.M.F. generated
(b) Cu losses
(c) output of the prime mover
(d) commercial, mechanical and electrical efficiencies.

$I_{sh} = 250/50 = 5A$

$I_a = 195 + 5 = 200$ A

a) $E_a = 250 + 200\times0.02 = 254$ V

b) $P_{cu\ a} = 200^2 \times 0.02 = 800$ W

$P_{cu\ sh} = 5^2 \times 50 = 1250$ W

c) Electromagnetic Power $P_{em} = E_a I_a = 254\times200 = 50800$ W

Mech. input power = output of prime mover = P_{em} + iron and friction losses

output of the prime mover = 50800 + 950 = 51750 W

$P_{out} = 250\times195 = 48750$ W

d) electrical Efficiency = P_{out} / P_{em} = 48750 / 50800 = 95.9647 %

mech. Efficiency = P_{em} / P_{mech} = 50800 / 51750 = 98.1643 %

overall (commercial) efficiency = 0.959647×0.981643×100 = 94.2031 %

Example:

A shunt generator has a F.L. current of 196 A at 220 V. The stray losses are 720 W and the shunt field coil resistance is 55 Ω. If it has a F.L. efficiency of 88%, find the armature resistance. Also, find the load current corresponding to maximum efficiency.

$I_{sh} = 220/55 = 4A$

$P_{cu\ sh} = 4^2 \times 55 = 880$ W

$I_a = 196 + 4 = 200$ A

$P_{out} = 220\times196 = 43120$ W

$P_{in} = 43120/0.88 = 49000$ W

Total losses = 49000 – 43120 = 5880 W

$P_{cu\ a} = 5880 - 880 - 720 = 4280\ \text{W} = 200^2 \times R_a$

$R_a = 0.107\ \Omega$

for maximum efficiency, the variable losses equal the constant losses.

Constant losses = stray loss + $P_{cu\ sh}$ = 720 + 880 = 1600 W

$I_a^2 \times 0.107 = 1600 \quad \rightarrow \quad I_a = 122.2836$ A

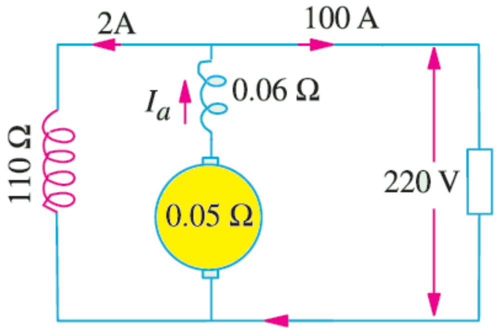
Example:

A long-shunt generator running at 1000 r.p.m. supplies 22 kW at a terminal voltage of 220 V. The resistances of armature, shunt field and the series field are 0.05, 110 and 0.06 Ω respectively. The overall efficiency at the above load is 88%. Find

(a) Induced voltage E_a

(b) Cu losses, iron and friction losses

(c) the torque exerted by the prime mover.



$I_{sh} = 220/110 = 2$ A, $I_t = 22000/220 = 100$ A,

$I_a = 100 + 2 = 102$ A,

$E_a = 102(0.05+0.06) + 220 = 231.22$ V,

$P_{cu\ a} = 102^2 \times 0.05 = 520.2$ W

$P_{cu\ s} = 102^2 \times 0.06 = 624.24$ W

$P_{cu\ sh} = 2^2 \times 110 = 440$ W

$P_{in} = 22000/0.88 = 25000$ W, $P_{losses} = 25000 - 22000 = 3000$ W

Iron & Friction Losses = 3000 – (520.2+624.24+440) = 1415.56 W

$P_{in} = 25000 = T\times\omega$, $\omega = 2\pi(1000/60) = 104.7198$

$T = 238.7324$ N.m

Example:

A short compound generator gives 240 V at a full-load output current of 100 A. The resistances of various windings of the machine are: armature 0.01 Ω, series field 0.2 Ω, shunt field 100 Ω. The iron loss, windage and friction losses are 1200 W. Calculate:

a) The efficiency of the machine at full load,
b) The efficiency of the machine at half load.

At Full Load I = 100A

Series winding drop = 100×0.2 = 20 V,

Voltage across shunt field winding = 240 + 20 = 260 V

I_{sh} = 260 / 100 = 2.6 A

I_a = 100 + 2.6 = 102.6 A

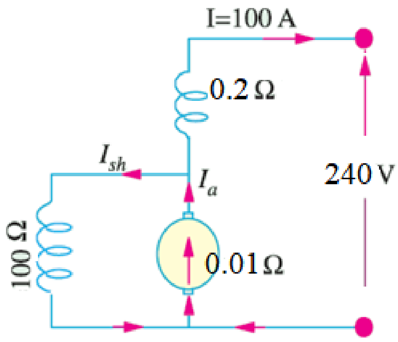
E_a = 240 + 100(0.2) + 102.6(0.01) = 261.026 V

Electromagnetic power $P_{em} = E_a I_a$ = 261.026 × 102.6 = 26781.2676 W

$P_{in} = P_{em}$ + stray loss = 26781.2676 + 1200 = 27981.2676 W

P_{out} = 240 × 100 = 24000 W

η = 24000 / 27981.2676 = 85.7717 %



At half full load I = 50A

Series winding drop = 50×0.2 = 10 V,

Voltage across shunt field winding = 240 + 10 = 250 V

I_{sh} = 250 / 100 = 2.5 A

I_a = 50 + 2.5 = 52.5 A

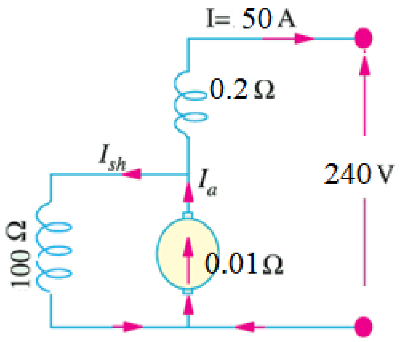
E_a = 240 + 50(0.2) + 52.5(0.01) = 250.525 V

Electromagnetic power $P_{em} = E_a I_a$ = 250.525 × 52.5 = 13152.5625 W

$P_{in} = P_{em}$ + stray loss = 13152.5625 + 1200 = 14352.5625 W

P_{out} = 240 × 50 = 12000 W

η = 12000 / 14352.5625 = 83.6088 %

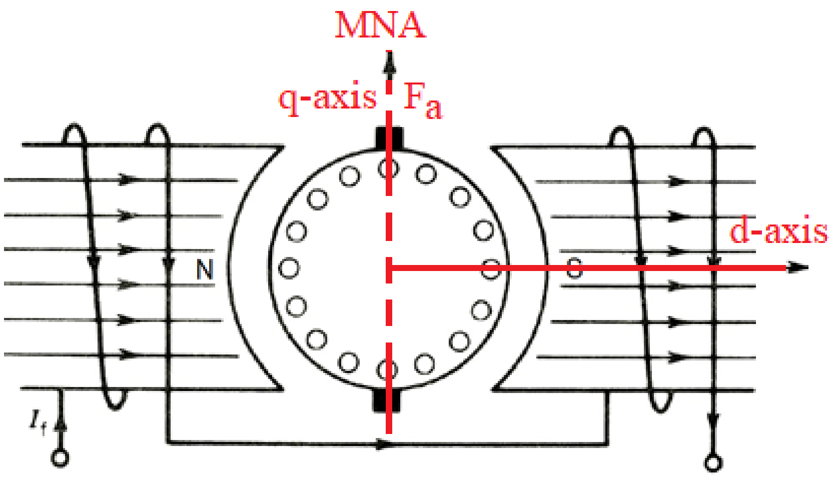


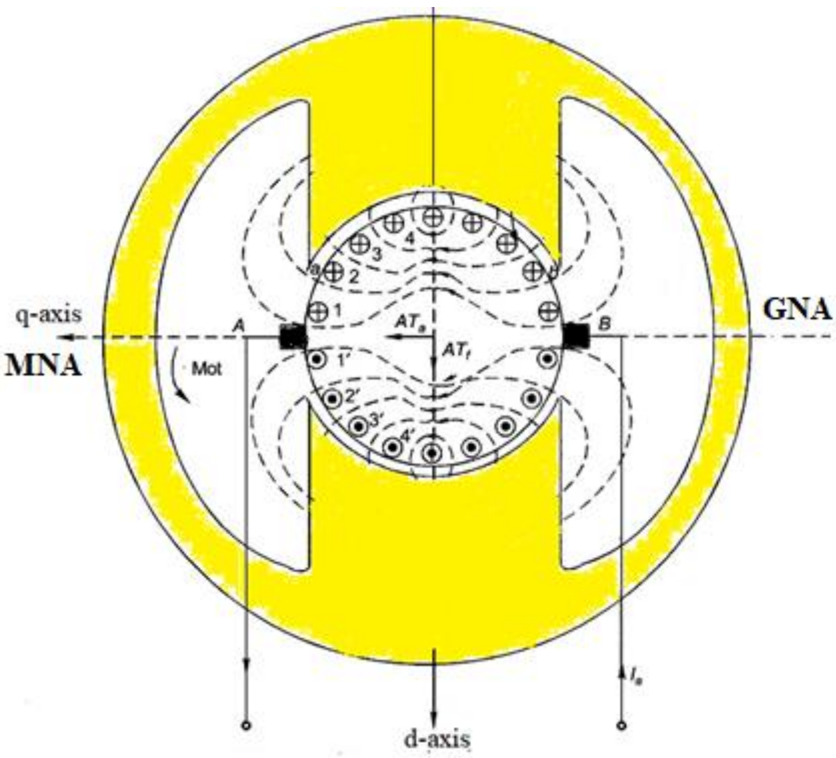
Armature Reaction (AR)

Geometrical Neutral Axis (GNA): is the axis that bisect the angle between the centre line of adjacent poles.

Magnetic Neutral Axis (MNA): is the axis drawn passing through brushes. Therefore, it sometimes called axis of commutation.

With no current flowing in the armature, the flux in the machine is established by the mmf produced by the field current acting along the d-axis, as shown in figure below. However, if the current flows in the armature circuit, it produces its own flux acting along the q-axis.

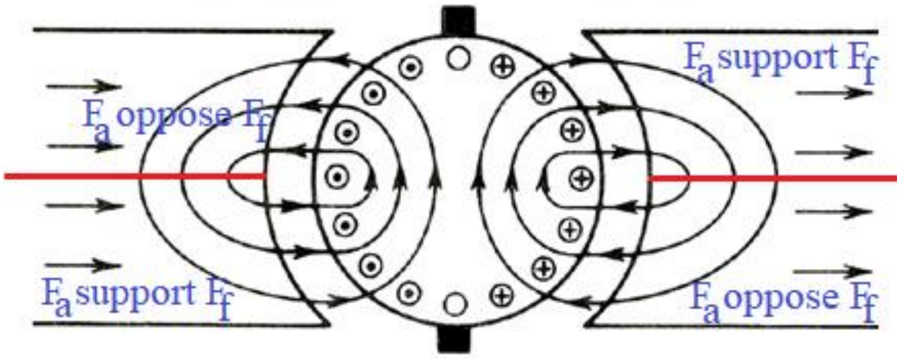




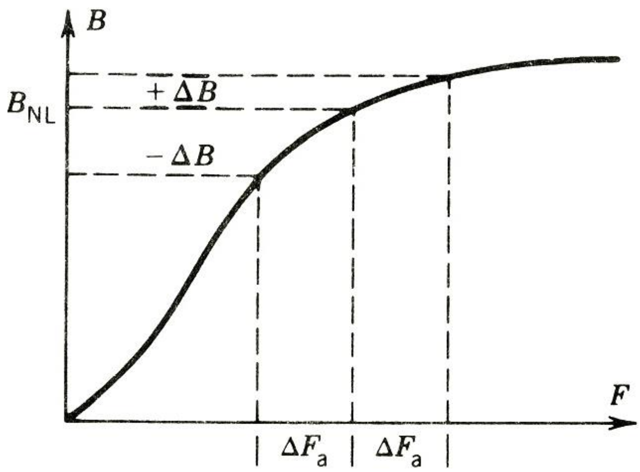
With no current in armature conductors, the MNA coincides with the GNA. However, when current flows in armature conductors, the combined action between the main flux and the armature flux shifts the MNA from GNA in the direction of rotation by certain angle (θ). This cause sparks at the contact points between brushes and commutator segments. To avoid these sparks, the MNA (where the brushes are located) must be shifted by the same angle in the direction of rotation. This is called armature reaction, it produces the following effects:

1- It weakens the main flux
2- It distorts the main flux

The original flux distribution in the machine due to the field current is disturbed. The armature flux opposes the field flux under one half of the pole and aids under the other half of the pole, as shown in figure below. Consequently, flux density under the pole increases in one half of the pole and decreases under the other half of the pole.

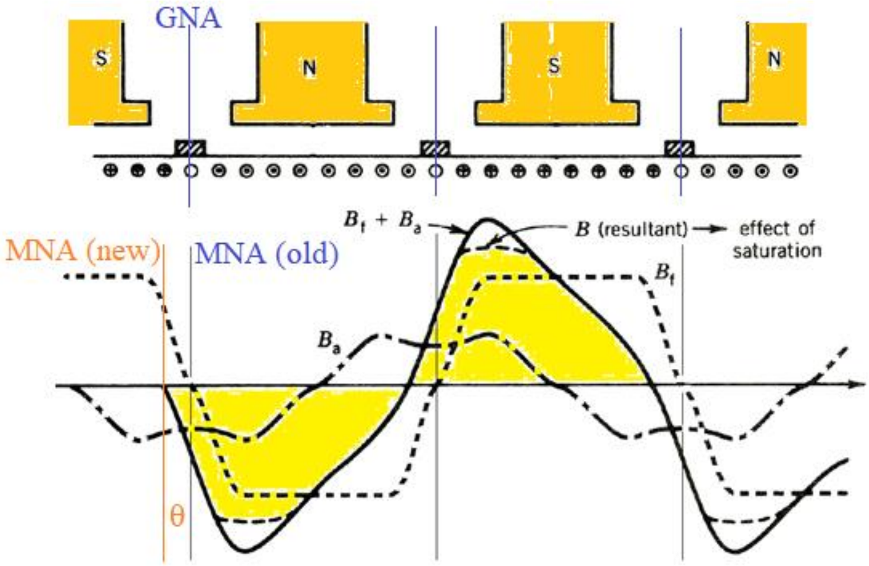


If the increased flux density causes magnetic saturation, the net effect is a reduction of flux per pole. This is illustrated in figure below.



For more explanation about AR, consider the developed diagram of Fig. a. The flux density distribution produced by the armature mmf (B_a), The flux density distribution produced by the field mmf (B_f) and the resultant flux density distribution ($B_f + B_a$) are given in Fig. b. The resultant flux density has a sawtooth waveform. From that figure, we notice that:

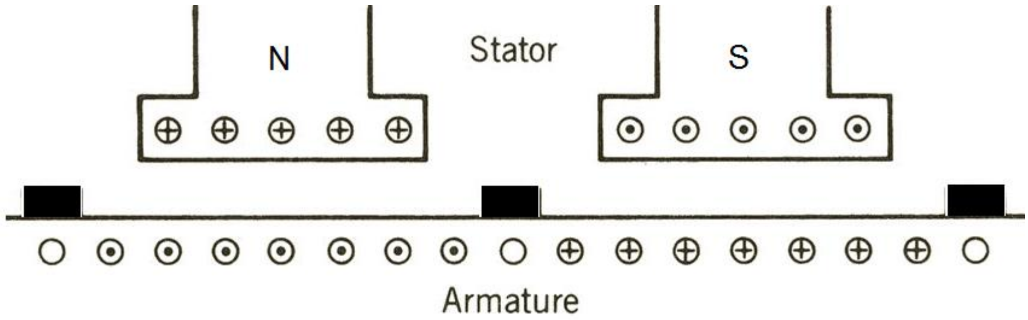
- Near one tip of a pole, the net flux density shows saturation effects (dashed portion).
- The zero-flux density region moves from the q-axis when armature current flows.
- If saturation occurs, the flux per pole decreases. This demagnetizing effect of armature current increases as the armature current increases.



The net effect of armature reaction can be considered as a reduction in the field current.

$$I_{f(eff)} = I_{f(actual)} - I_{f(AR)}$$

Moreover, the armature mmf distorts the flux density distribution and also produces the demagnetizing effect known as armature reaction. The zero-flux density region shifts from the q-axis because of armature mmf, and this causes poor commutation leading to sparking. Much of the rotor mmf can be neutralized by using a compensating winding, which is fitted in slots cut on the main pole shoe. These pole shoe windings are so arranged that the mmf produced by currents flowing in these windings opposes the armature mmf. This is shown in the developed diagram of figure below.



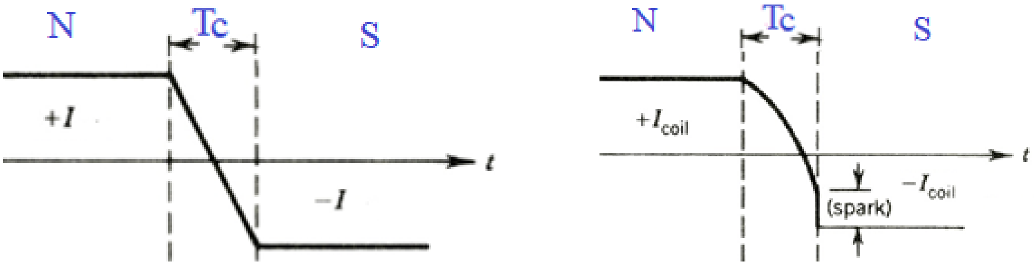
The compensating winding is connected in series with the armature winding so that its mmf is proportional to the armature mmf. The slotted pole shoe windings are shown in figure below.

Commutation Process:

The current induced in armature conductors of a DC generator is alternating. This means, the current flows in one direction when armature conductors are under *N*-pole and in the opposite direction when they are under *S*-pole. To make armature current flows unidirectional in the external circuit, a commutator is used. As conductors cross an *N*-pole and enter under *S*-pole, the current is reversed. This occurs along the Magnetic Neutral Axis (MNA) or brush axis.

The brush width is equal to the width of one commutator segment. When the brush covers two adjacent commutator segments, one coil is short circuited and its current undergoing reversal from +I to −I. This process by which current in the short-circuited coil is reversed while it crosses the MNA is called commutation. The brief period during which coil remains short-circuited is known as commutation period Tc.

If the current changed from $+I$ to zero to $-I$ is completed by the end of T_c, then the commutation is ideal. If current reversal is not completed by that time, then spark occurs between brush and the commutator which results in progressive damage to both.

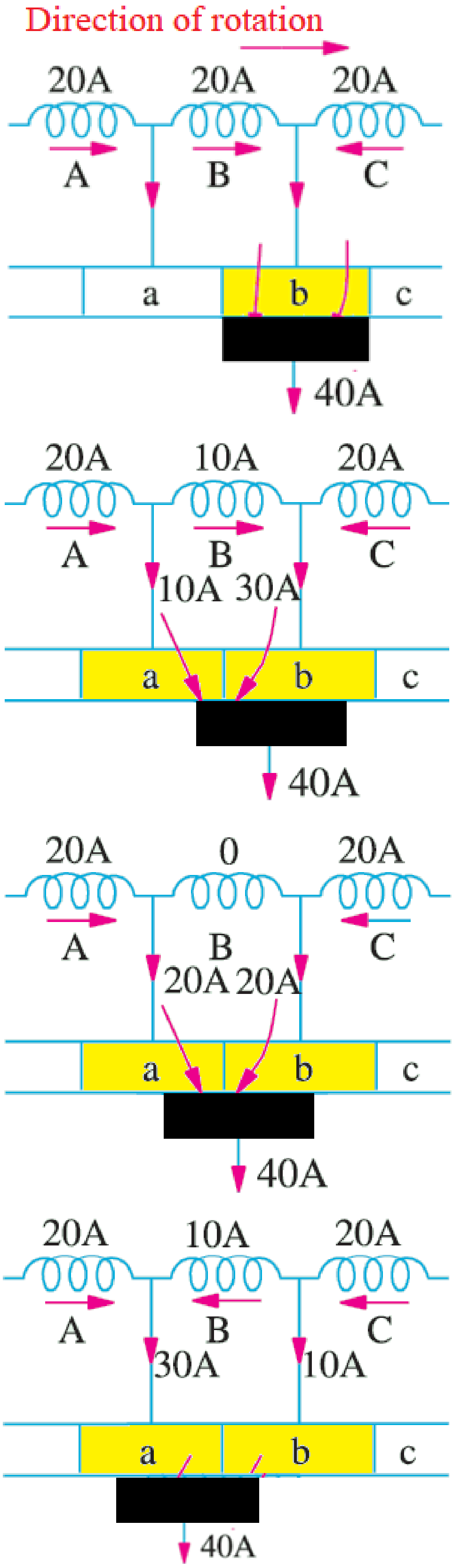


Consider a three armature coils A, B and C each carry a current of 20A. Now the brush is in touch with segment '*b*' and the brush current is 40A (20 A from coil B and 20 A from coil C) as illustrated in Fig a.

As the armature rotate in the direction indicated, coil B is about to be short circuited because brush is about to come in touch with commutator segment '*a*'. The current through coil B is reduced from 20 A to 10 A because the other 10 A flows via segment '*a*'. As the contact area of the brush is more with segment '*b*' than with segment '*a*', it receives 30 A from the previous coil C, the total again is 40 A as shown in Fig. b.

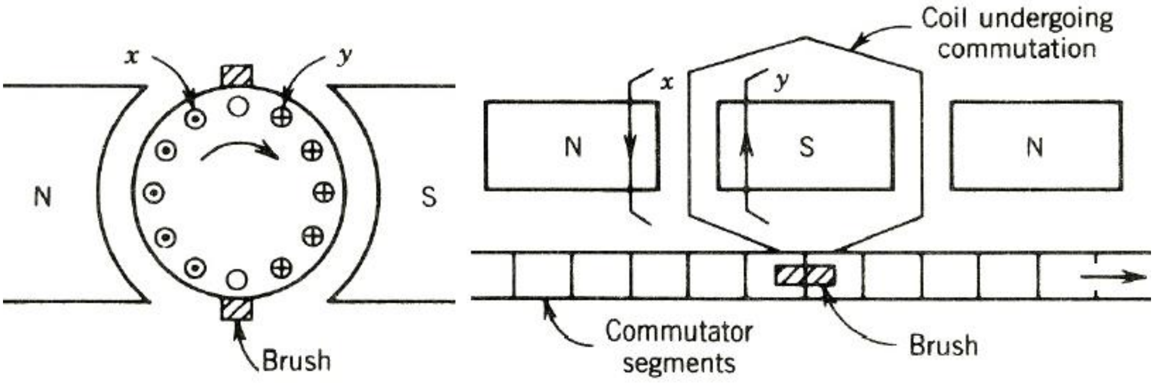
As the armature continue rotating, the coil B becomes in the middle of its short-circuit period, and its current is decreased to zero. The two currents of value 20 A each, pass to the brush directly from coil *A* and *C* as shown in Fig. c. The brush contact areas with the two segments '*b*' and '*a*' are equal.

As the armature continue rotating, brush contact area with segment '*b*' is decreasing whereas that with segment '*a*' is increasing. Coil *B* now carries 10 A in the reverse direction which combines with 20 A supplied by coil *A* to make up 30 A that passes from segment '*a*' to the brush. The other 10 A is supplied by coil *C* and passes from segment '*b*' to the brush, again giving a total of 40 A at the brush as shown in Fig. d.

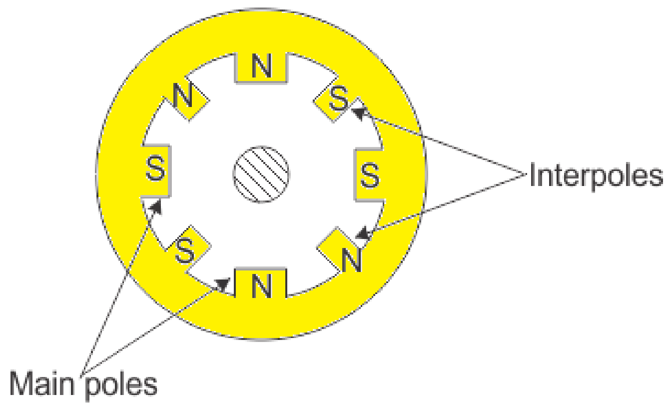

Direction of rotation
20A
20A
20A
A
B
C
a
b
c
40A
20A
10A
20A
A
B
C
10A 30A
a
b
c
40A
20A
0
20A
A
B
C
20A 20A
a
b
c
40A
20A
10A
20A
A
B
C
30A
10A
a
b
c
40A

Interpoles or Commutator Poles:

The purpose of commutators and brushes in DC machine is to reverse the current in a conductor when it goes from one pole to the next. When the conductor x is under the north pole, it carries a dot current, but after passing through the brush it comes under the south pole (conductor y) and thus carries the cross current.



In the developed diagram shown, when the coil passes the brush, its current changes direction. Ideally the current changes linearly in the coil. This ideal situation provides a smooth transfer of current. But due to coil inductance, the actual current through a coil undergoing commutation is shown in Fig. below. When the coil is about to leave the brushes, the current reversal is not complete. Therefore, the current has to jump to its full value almost instantaneously, and this will cause sparking. To improve commutation, a small pole, called an interpole or commutation pole, is created. Its winding carries the armature current in such a direction that its flux opposes the q-axis flux produced by armature current flowing in the armature winding. As a result, the net flux in the interpolar region is almost zero.

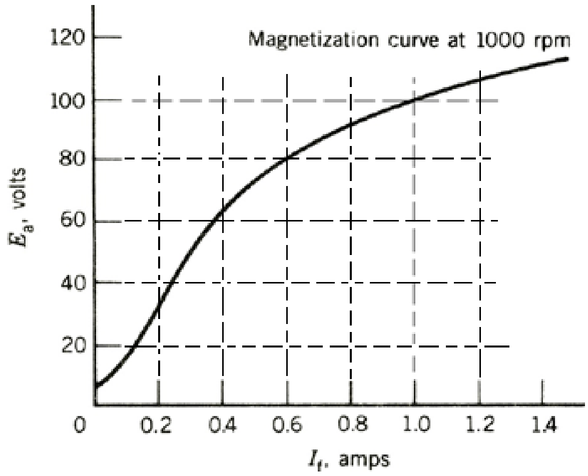


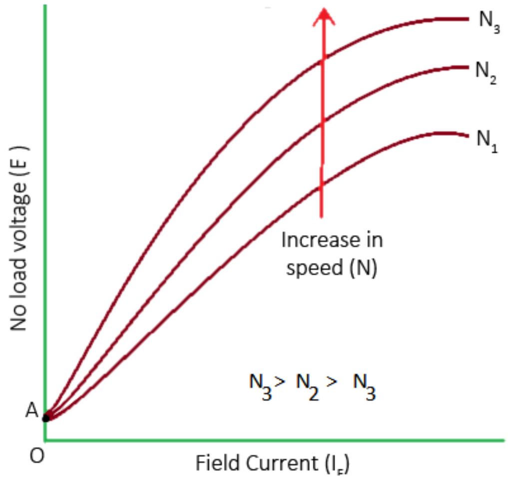
DC Generator Characteristics

There are three types of characteristics:

No-load characteristics: (E_a Versus I_f)

It is also known as Open-Circuit characteristics (O.C.C). Its shape is practically the same for all generators whether separately-excited or self-excited.





As seen from the above characteristic, the generated e.m.f E_a is directly proportional to the field current as well as the generator speed. However, even when the field current is zero, some amount of emf is generated (OA). This initially induced emf is due to residual magnetism in the field poles. Therefore, a small initial emf is induced in the armature. The first part of the O.C.C. follows a straight line. This means, as the flux increases the induced voltage increases with the same amount. For further increase in the field current, the poles get saturated and the flux becomes practically constant. Thus, E_a also remains constant. Hence, the O.C.C. curve looks like the B-H characteristic.

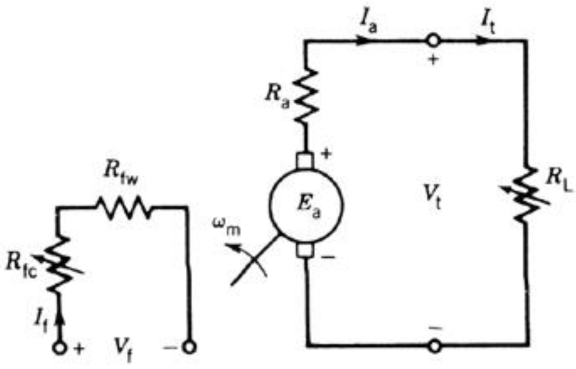
Internal (Total) Characteristic (E_a Versus I_a)

It gives the relation between E_a and the armature current I_a. This characteristic is of interest mainly to the designer.

External Characteristic (V_t Versus I_t)

It is referred to as voltage-regulating curve. It gives relation between that terminal voltage V_t and the load current I_t. The values of V_t are obtained by subtracting I_aR_a from corresponding values of E_a. This characteristic is of great importance in judging the suitability of a generator for a particular purpose.

1-Separately-Excited DC Generator



$$V_f = R_f I_f$$

$$R_f = R_{fw} + R_{fc}$$

$$E_a = V_t + I_a R_a$$

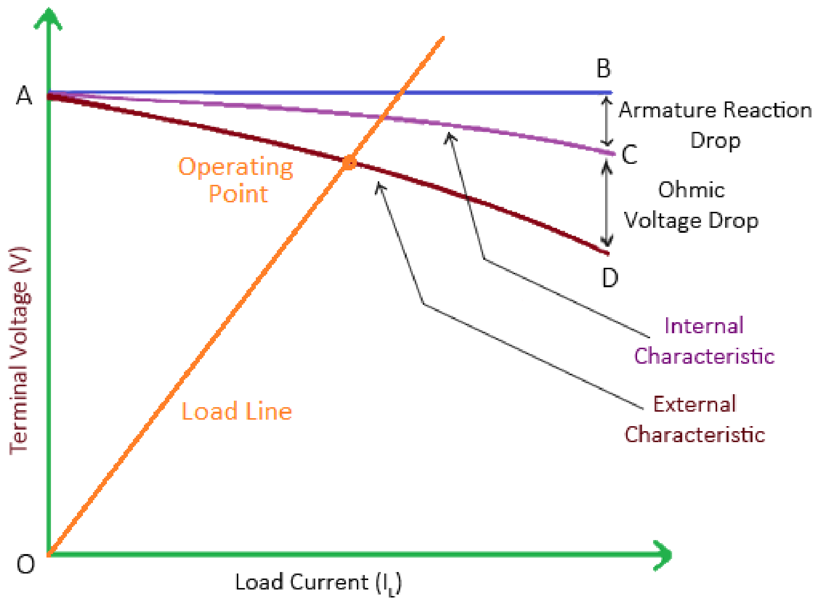
$$V_t = E_a - R_a I_a$$

$$E_a = K_a \Phi \omega_m$$

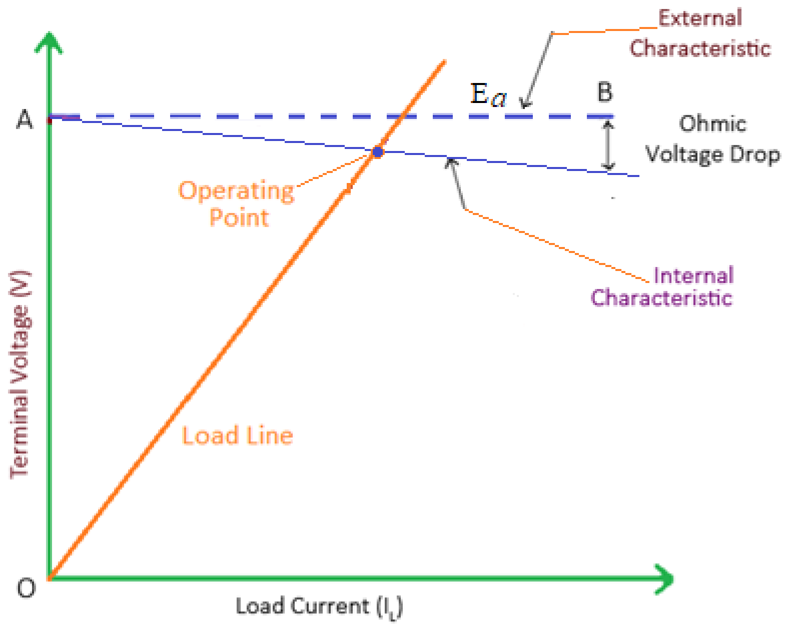
$$V_t = I_t R_L$$

$$I_a = I_t$$

The external characteristic of the separately excited DC generator, considering the armature reaction, is shown below. The straight-line AB in above figure represents the no-load voltage E_a vs load current I_L. Due to the demagnetizing effect of armature reaction, the generated emf at load is less than that at no-load. The curve AC represents the loaded emf E_a vs I_L (internal characteristic as $I_a = I_L$ for a separately excited). Also, the terminal voltage is lesser due to ohmic drop occurring in the armature and brushes. The curve AD represents the terminal voltage vs load current (external characteristic). The load characteristic is plotted as straight line with slope R_L. The point of intersection between the generator external characteristic and the load characteristic determines the operating point.



If there is no armature reaction and armature voltage drop, the external characteristic is as shown below.



Example:

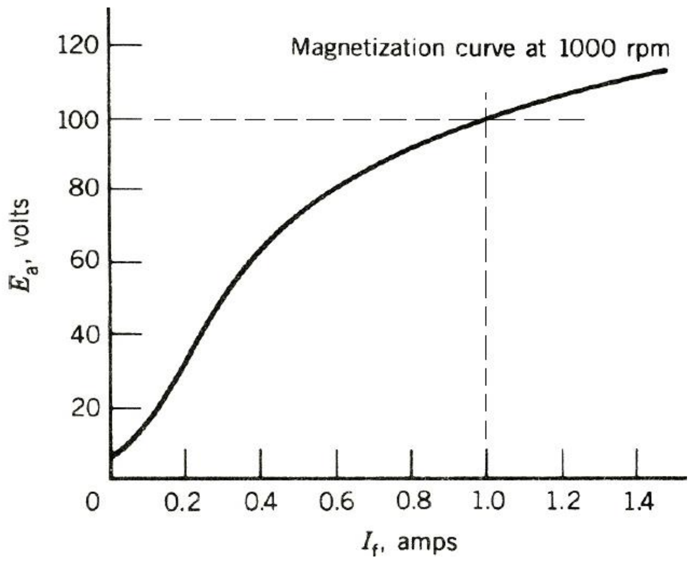
A 12 kW, 100 V, 1000 rpm separately-excited DC generator has armature resistance R_a =0.1 Ω, field winding resistance R_{fw} =80 Ω.The rated field current is 1 A. The magnetization characteristic (O.C.C.) at 1000 rpm is shown below.

(a) Neglect the armature reaction effect. Determine the terminal voltage at full load.

(b) Consider that armature reaction at full load is equivalent to 0.06 field amperes.

(i) Determine the full-load terminal voltage.

(ii) Determine the field current required to make the terminal voltage V_t =100 V at full-load condition.



(a) $V_t = E_a - I_a R_a$
$= 100 - 120 \times 0.1$
$= 88$ V

(b) **(i)** From Eq. $I_{f(eff)} = I_{f(actual)} - I_{f(AR)}$

$I_{f(eff)} = 1 - 0.06 = 0.94$ A

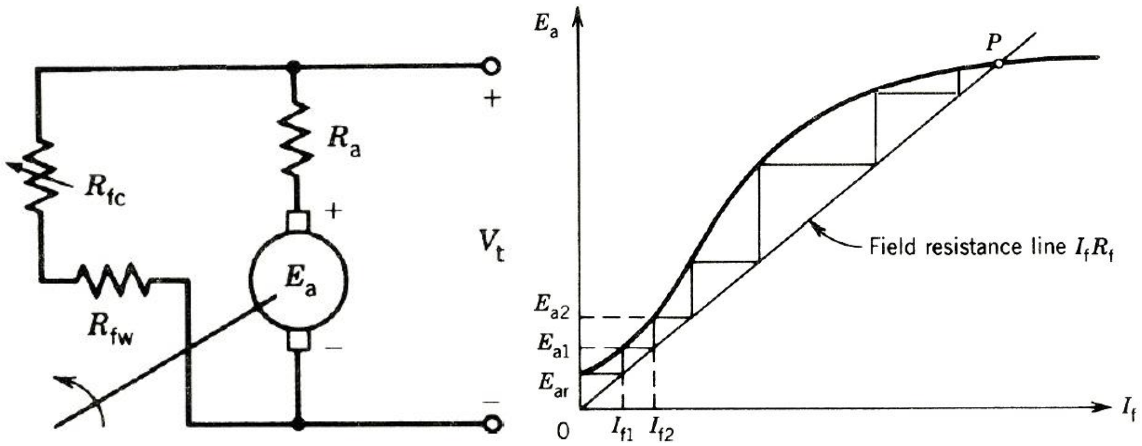
From o.c.c at this field current,
$E_a = 98$ V

$V_t = E_a - I_a R_a = 98 - 120 \times 0.1 = 86$ V

(ii) $E_a = V_t + I_a R_a = 100 + 120 \times 0.1 = 112$ V
From o.c.c., the effective field current required is $I_{f(eff)} = 1.4$ A

$I_{f(actual)} = 1.4 + 0.06 = 1.46$ A

2-Shunt DC Generator



$$V_t = I_f R_f$$

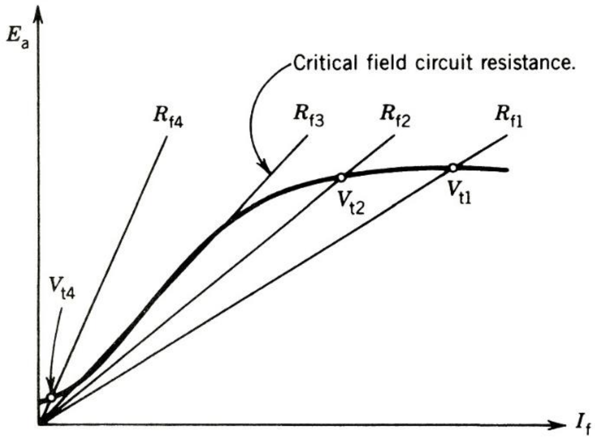
$$R_f = R_{fc} + R_{fw}$$

$$E_a = I_a R_a + V_t$$

$$I_t = I_a - I_f$$

Some residual magnetism must exist in the magnetic circuit of the generator. The magnetization curve of the DC machine is shown above. Also the field resistance line, which is a plot of $I_f R_f$ versus I_f is shown in this figure. The voltage build-up process in the shunt DC generator is as follows.

When the armature is driven at a certain speed, a small voltage, E_{ar}, will appear across the armature terminals because of the residual magnetism in the machine. Therefore, a current I_{f1} will flow in the field winding. With I_{fl} flowing in the field circuit, the generated voltage is E_{al} but the terminal voltage is $V_t = I_{fl}R_f < E_{al}$. The increased armature voltage E_{al} will eventually increase the field current to a larger value I_{f2}, which in turn will build up the armature voltage to E_{a2}. This process of voltage build-up continues. If the voltage drop across R_a is neglected (as $R_a << R_f$), the voltage builds up to the value given by the crossing point (P) of the magnetization curve and the field resistance line. At this point, $E_a = I_fR_f = V_t$ (assume Ra is neglected), and no excess voltage is available to further increase the field current. In the actual case, the changes in I_f and E_a take place simultaneously and the voltage build-up follows approximately the magnetization curve. Figure below shows the voltage build-up for different R_f. At some resistance value R_{f3}, the resistance line is almost coincident with the linear portion of the magnetization curve, producing an unstable voltage situation. This resistance is known as the critical field circuit resistance. If the resistance is greater than this value, such as R_{f4}, build-up (V_{t4}) will be insignificant. On the other hand, if the resistance is smaller than this value, such as R_{f1} or R_{f2}, the generator will build up higher voltages (V_{t1},V_{t2}).



Conclusion:

1. Residual magnetism must be present in the magnetic system.
2. Field winding mmf should aid the residual magnetism.
3. Field circuit resistance should be less than the critical field circuit resistance.

Example:

The magnetization curve of a DC shunt generator at 1500 r.p.m. is:

I_f (A) :	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_0 (V) :	6	60	120	172.5	202.5	221	231	237	240

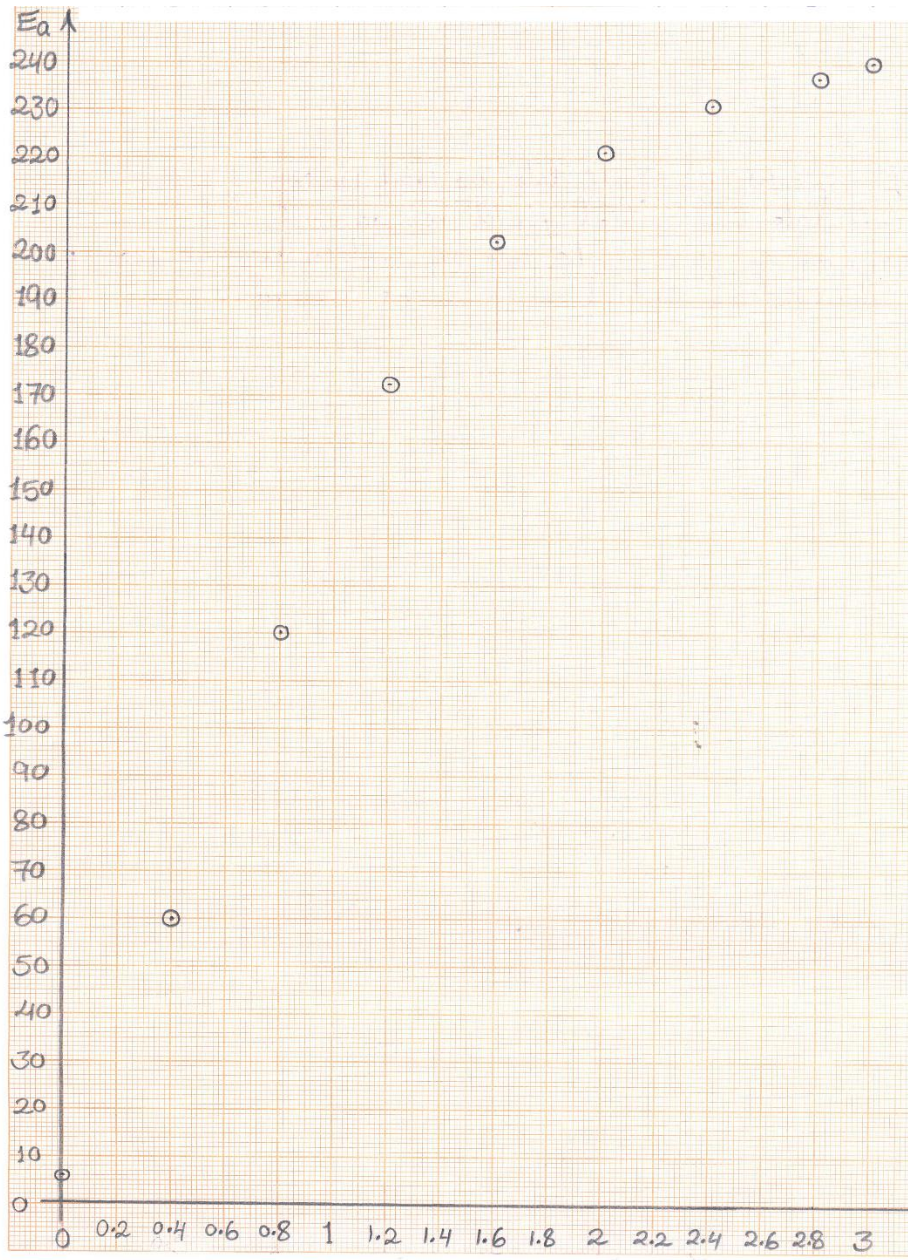
For this generator, find:

(i) No load e.m.f. for a total shunt field resistance of 100 Ω

(ii) The critical field resistance at 1500 r.p.m.

(iii) The magnetization curve at 1200 r.p.m. and the open-circuit voltage for a field resistance of 100 Ω.

The magnetisation curve at 1500 rpm is plotted below from the given data.



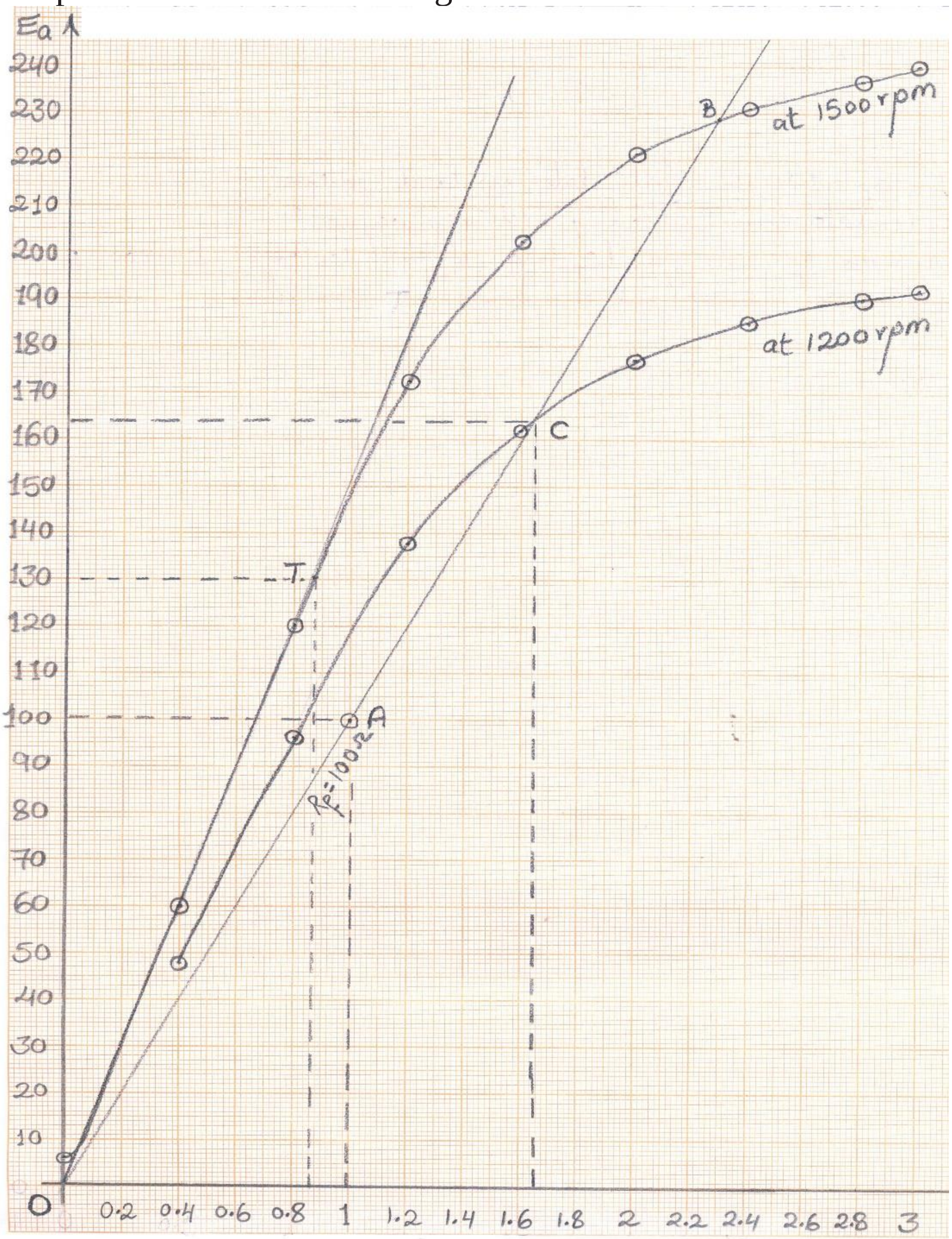
i) The 100 Ω resistance line *OA* is obtained by joining the origin (0, 0) with the point (1A, 100 V). This line intersects the OCC at point B. The voltage corresponding to point *B* is 227.5 V. Therefore, the no-load voltage to which generator will build-up is 227.5 V.

ii) From the origin (0,0), draw a tangent to the OCC. The tangent *OT* represents the critical resistance at 1500 rpm. At that point E_a = 130 V, I_f = 0.86 A. R_f = 130/0.86 = 151.163 Ω.

iii) For 1200 r.p.m., the induced voltages for different field currents would be (1200/1500) = 0.8 of those for 1500 r.p.m. The values of these voltages are tabulated below:

$I_f(A)$:	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_0\,(V)$:	4.8	48	96	138	162	176.8	184.8	189.6	192

The new OCC curve is also plotted. The 100 Ω line intersects the curve at point *C* which corresponds to an induced voltage of 164 V.



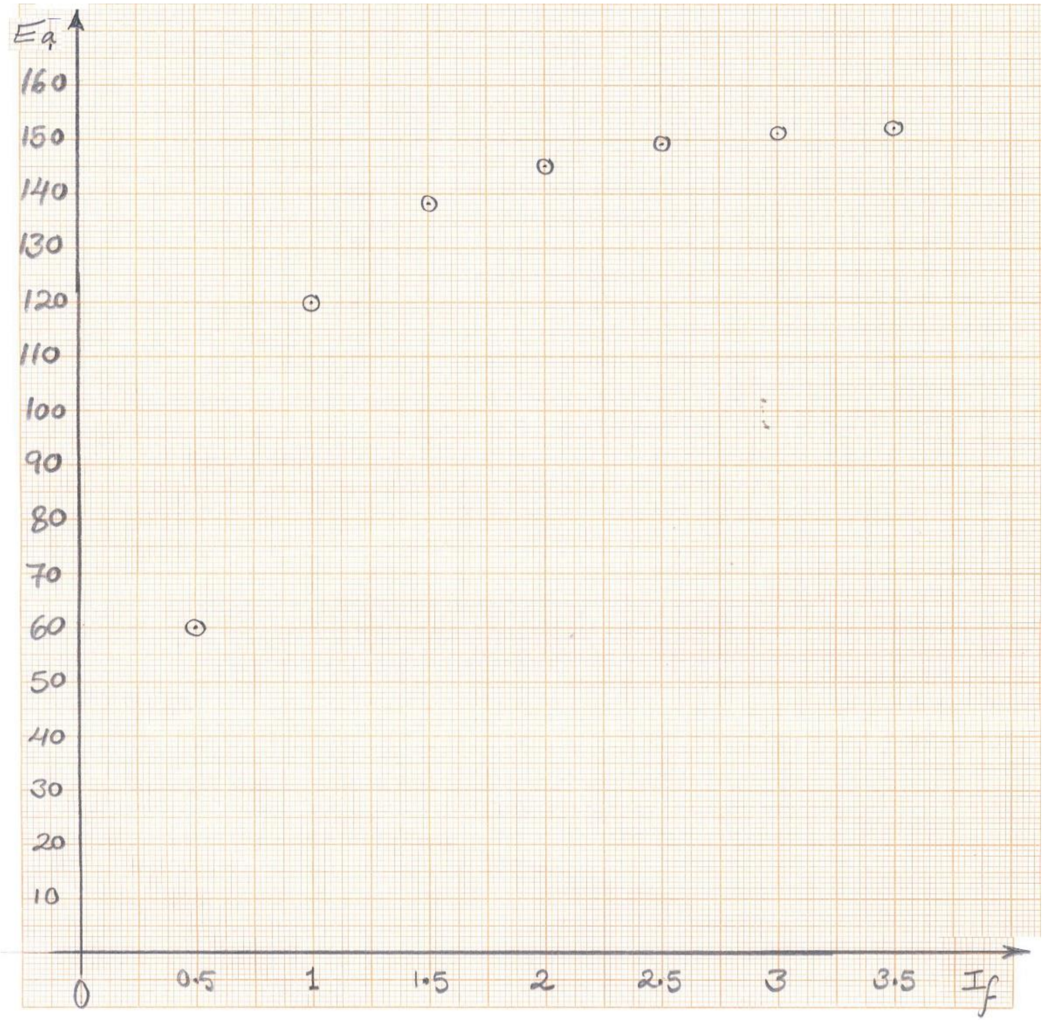
Example:

The O.C characteristic of a DC shunt generator driven at rated speed is as follows:

Field Amperes :	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*	*3.5 A*
Induced Voltage :	*60*	*120*	*138*	*145*	*149*	*151*	*152 V*

If resistance of field circuit is adjusted to 53 Ω, calculate:

a) The open circuit voltage,
b) The load current at terminal voltage of 100 V. Neglect armature reaction and assume an armature resistance of 0.1 Ω.



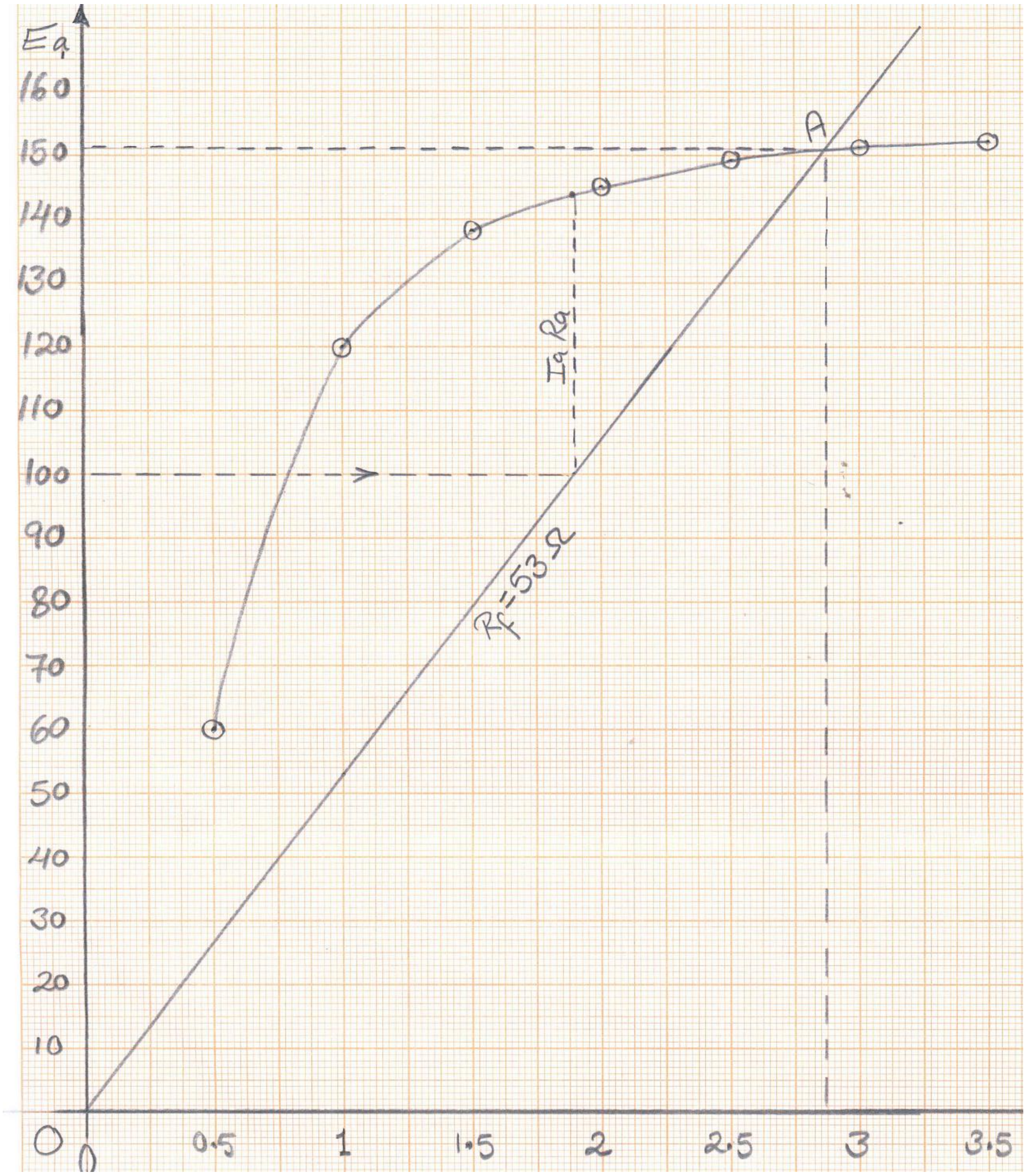
At R_f = 53 Ω, the line OA is drawn to intersect the OCC. At the point A, the terminal voltage will be 151 v,

Now, at V = 100 V, $I_{sh} = I_f$ = 100/53 = 1.89 A

Generated voltage corresponding to this exciting current as seen from graph is 144 V.

Now $E_a = V + I_a R_a$ or $I_a R_a$ = 144 − 100 = 44 V ∴ 0.1 I_a = 44 or I_a = 44/0.1 = **440 A**

Load current = $I_a + I_{sh}$ = 440 + 1.89 = 441.89 A



Example:

A 12 kW, 100 V, 1000 rpm DC shunt generator has armature resistance R_a =0.1 Ω, shunt field winding resistance R_{fw} =80Ω. The rated field current is 1.0 A. The magnetization characteristic at 1000 rpm is shown. The machine is operated at no load.

a) When the field circuit control resistance (R_{fc}=0), determine the maximum value of the generated voltage.

b) Determine the value of (R_{fc}) required to generate rated terminal voltage.

c) Determine the value of the critical field circuit resistance.

a) Draw field resistance line for R_f = 80Ω, from the origin (0, 0) to the point (I_f = 1 A, I_fR_f = 80). The intersection of this line with magnetizing curve of the machine determine the maximum generated voltage, which is E_a = 111 V.

b) Draw a field resistance line that intersects O.C.C curve at 100 V. For this case,

$$I_f = 1 \text{ A}$$

$$R_f = \frac{100}{1} = 100\ \Omega = R_{fw} + R_{fc}$$

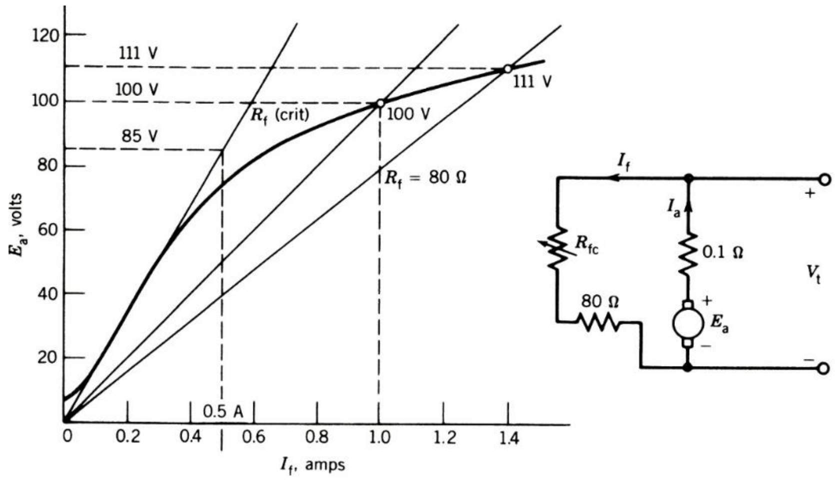
$$R_{fc} = 100 - 80 = 20\ \Omega$$

Draw the critical field resistance line (tangent to the linear portion of the magnetization Curve and passing through origin). From this line:

$$\text{For } I_f = 0.5, E_a \text{ is } 85 \text{ V.}$$

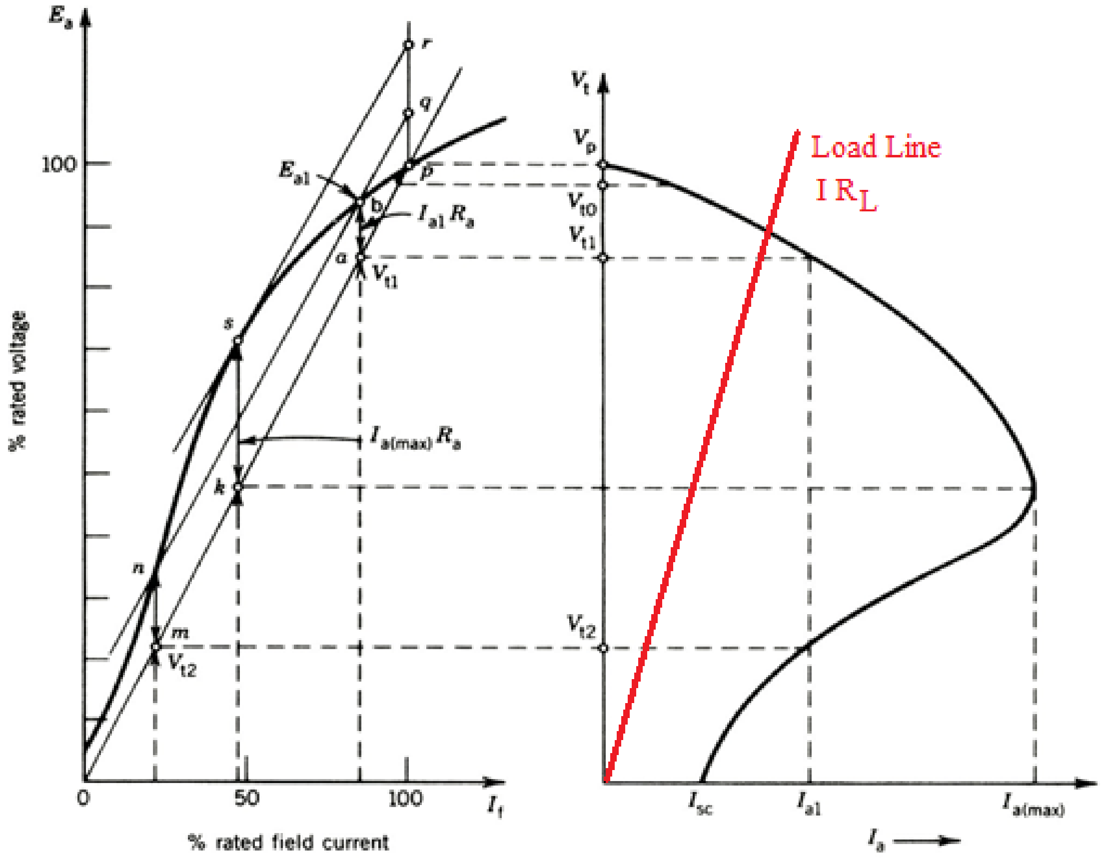
$$R_{f(crit)} = \frac{85}{0.5} = 170\ \Omega$$

$$R_{fc} = 170 - 80 = 90\ \Omega$$



Voltage-Current characteristics of DC shunt generator (External Characteristics)

The vertical distance between the magnetization curve and the field resistance line represents the I_aR_a voltage drop. Consider the various points on the field resistance line, which also represents the terminal voltage V_t. For each terminal voltage, such as V_{t1}, compute the armature current I_{a1} from the I_aR_a voltage drop, which is the vertical distance between V_{t1} and E_{a1}. If this calculation is performed for various terminal voltages, the voltage–current characteristic of the DC shunt generator will be obtained.



A convenient way to construct the voltage–current characteristic from the magnetization curve and field resistance line is to draw a vertical line at point P. This vertical line represents the I_aR_a drop. For example, the vertical line *pq* represents I_aR_a drop. A line *qbn* is drawn parallel to *op*. Therefore, *pq*=*ab*=*mn*=$I_{a1}R_a$. The same armature current results in two terminal voltages, V_{t1} and V_{t2}. To obtain the value of the maximum armature current that can be drawn from the DC generator, a line *rs* is drawn // *op* and tangential to the magnetization curve. This will result in maximum vertical distance, *sk*. Also if the machine terminals are shorted (i.e., R_L= 0), the field current is zero and the machine current (I_a =I_t = E_{ar}/Ra) which is not very high.

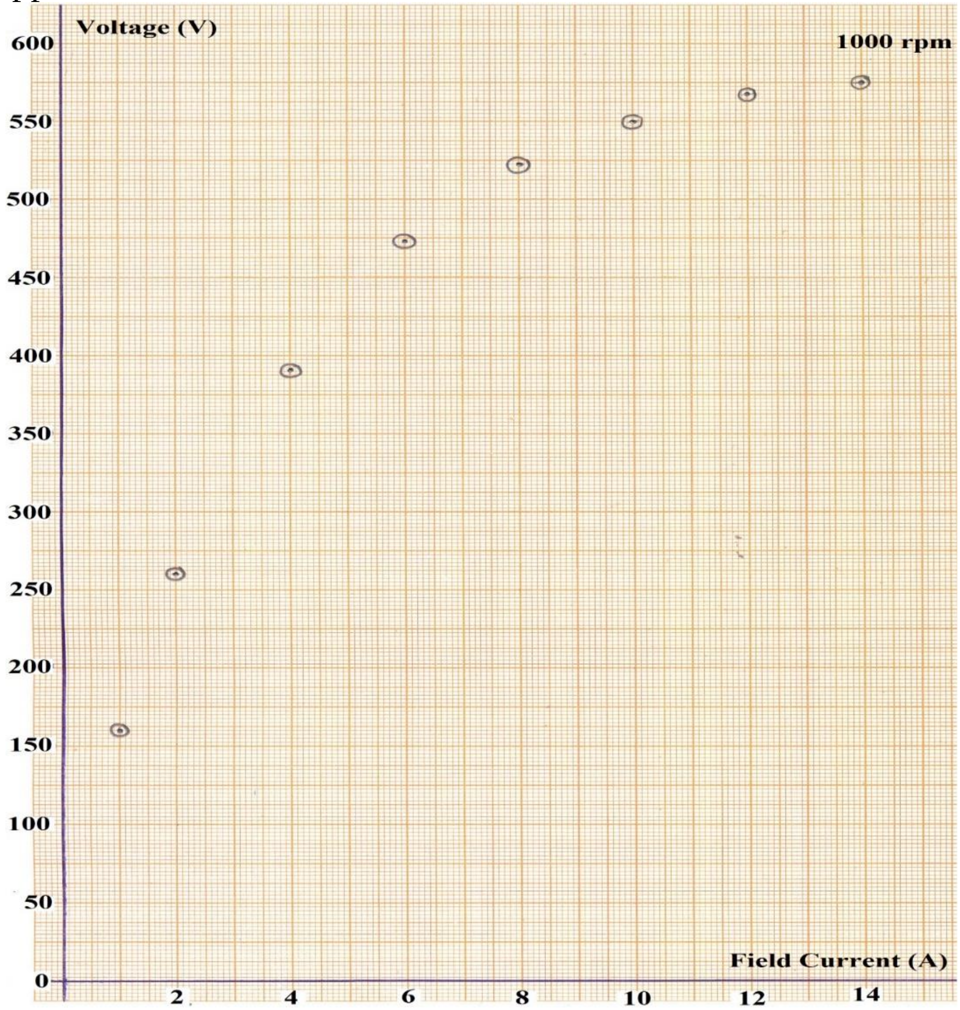
Example:

The Open Circuit Characteristic (O.C.C) of a DC shunt generator has the following readings that taken at 1000 rpm:

Field Current (A)	1	2	4	6	8	10	12	14
Induced Voltage (V)	160	260	390	472	522	550	567	575

In order to save your time, the O.C.C. is plotted and attached (next page).
If the field resistance is adjusted to 50 Ω and armature resistance is 0.4 Ω, calculate:

a) The voltage to which the generator will build-up on open circuit,
b) The maximum possible armature current,
c) The speed to generate 450V on no-load, at the same field current obtained in (a),
d) The field resistance required to generate 500 V on no-load at 1000 rpm,
e) The approximate value of the critical resistance of shunt circuit.



a) At R_f = 50 Ω, draw a straight line I_fR_f, this line intersects the OCC at point A. at this point:

V = 562.5 V I_f = 11.2 A

b) Draw a line // I_fR_f and tangent the OCC at point B. at that point I_aR_a = 190 V
Then I_a max = 190/0.4 = 475 A
c) At I_f = 11.2 (from a) the OCC voltage = 562.5 at 1000 rpm
But it is required that OCC voltage = 450 V, then

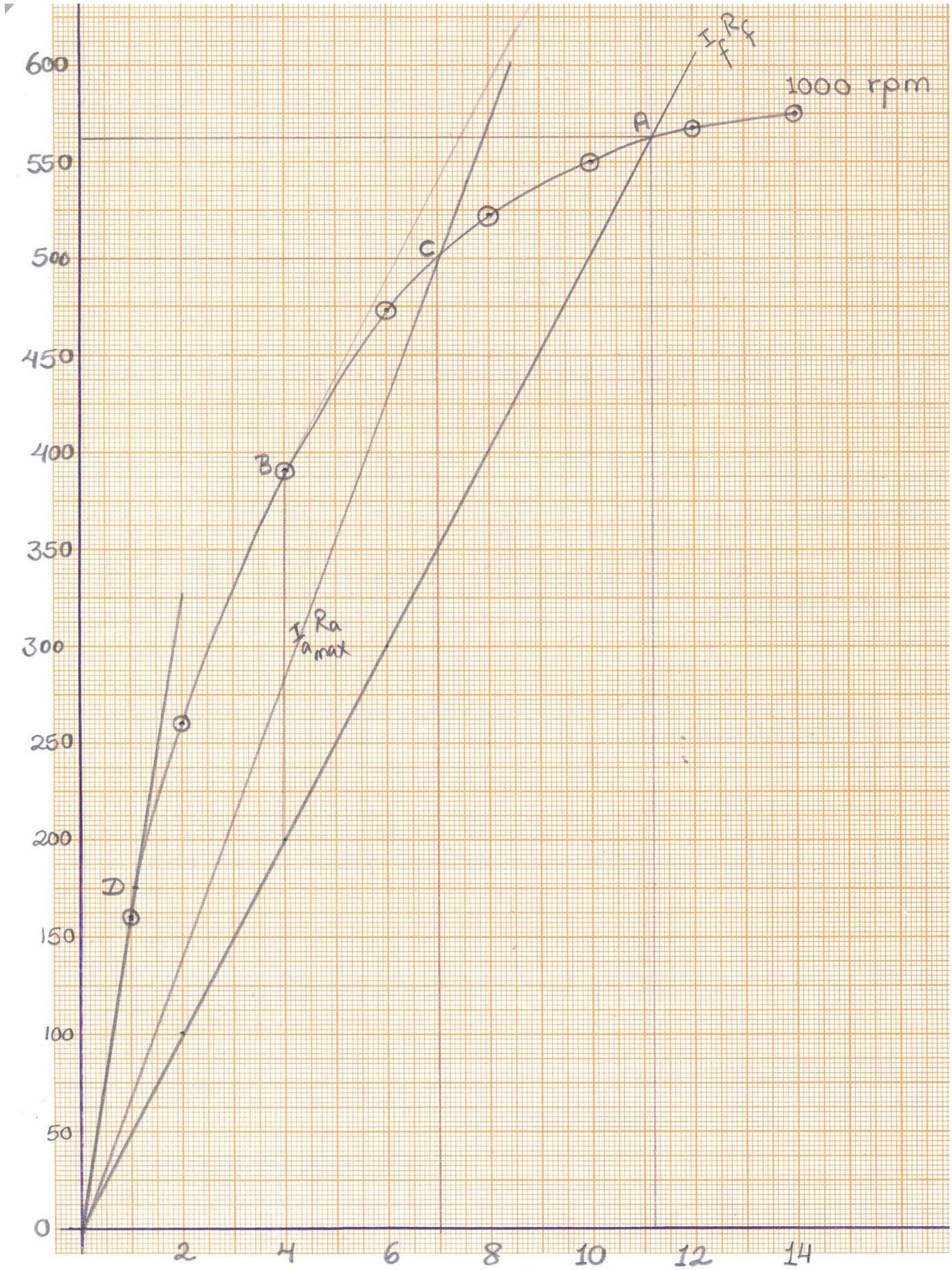
$$\frac{562.5}{450} = \frac{1000}{N} \quad \rightarrow \quad N = 800\ rpm$$

d) It is required to generate 500 V at 1000 rpm, therefore we draw a line I_fR_f that intersect the OCC at that point C. it is found that V = 500, I_f = 7

$$R_f = \frac{500}{7} = 71.43\ \Omega$$

e) The critical resistance is obtained by drawing the line I_fR_f that tangent OCC at point D. at that point V=175 V, I_f = 1.1 A

$$R_f critical = \frac{175}{1.1} = 159.091\ \Omega$$



Example:

The Open Circuit Characteristic (O.C.C) of a DC shunt generator has the following readings that taken at 300 rpm:

Field Current (A)	0	1	2	3	4	5	6	7
Induced Voltage(V)	8	38	92	132	160	179	190	195

To save your time, the O.C.C. is plotted and attached

If the field resistance is 30 Ω and armature resistance is 0.2 Ω, calculate:

a) The voltage to which the generator will build-up on open circuit,
b) The maximum possible armature current,
c) Fill in the spaces of the following table then sketch the external characteristics,

Field Current (I_f)	0	1	2	3	4	5	6
Induced Voltage (E_a)	………	………	………	………	………	………	………
Terminal Voltage (V_t)	………	………	………	………	………	………	………
Armature Current (I_a)	………	………	………	………	………	………	………
Terminal Current (I_t)	………	………	………	………	………	………	………

d) The terminal voltage and load current for a load resistance of 1.5 Ω,
e) The approximate value of the critical resistance of shunt circuit.

At R_f = 30 Ω, draw a straight line I_fR_f, this line intersects the OCC at point A. at this point: V = 193 V I_f = 6.5 A

Draw a line // I_fR_f and tangent the OCC at point B where I_f = 3.5A. at that point I_aR_a = 43 V Then I_a max = 43/0.2 = 215 A

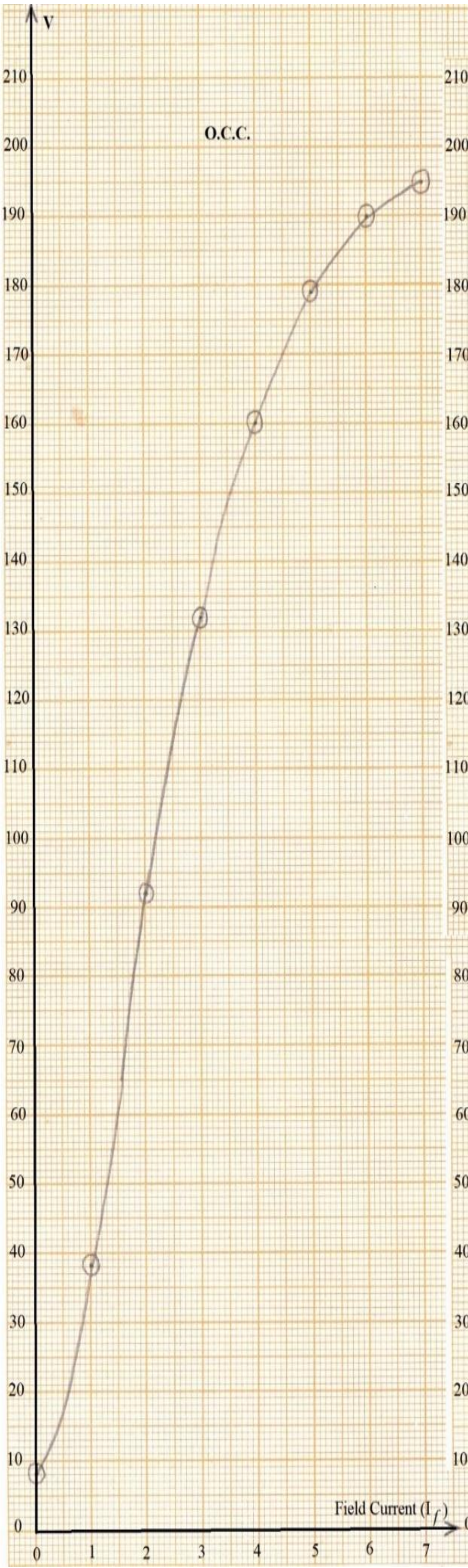
Fill the table as follows:

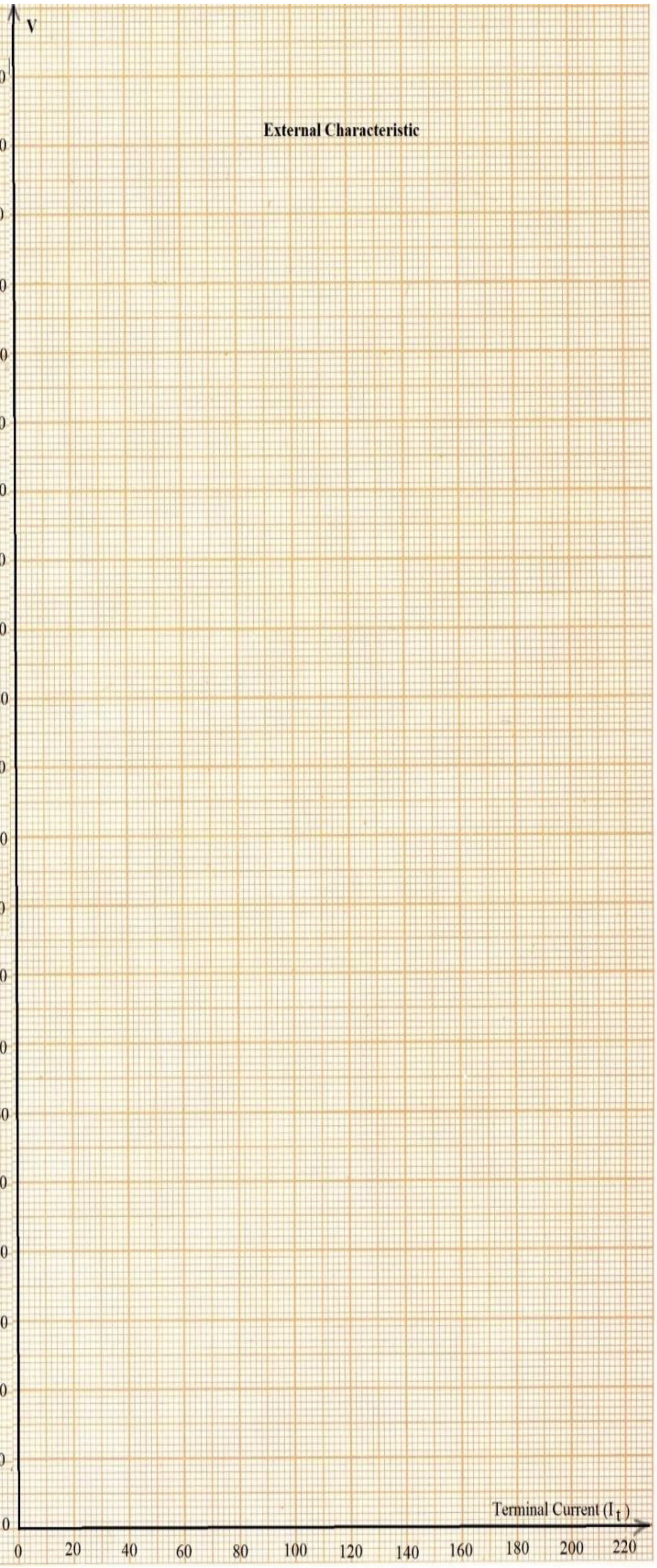
Field Current (I_f)	0	1	2	3	3.5	4	5	6
Induced Voltage (E_a)	8	38	92	132	148	160	179	190
$V_t = I_f R_f$	0	30	60	90	105	120	150	180
$I_a = (E_a - V_t)/R_a$	40	40	160	210	215	200	145	50
$I_t = I_a - I_f$	40	39	158	207	211.5	196	140	44

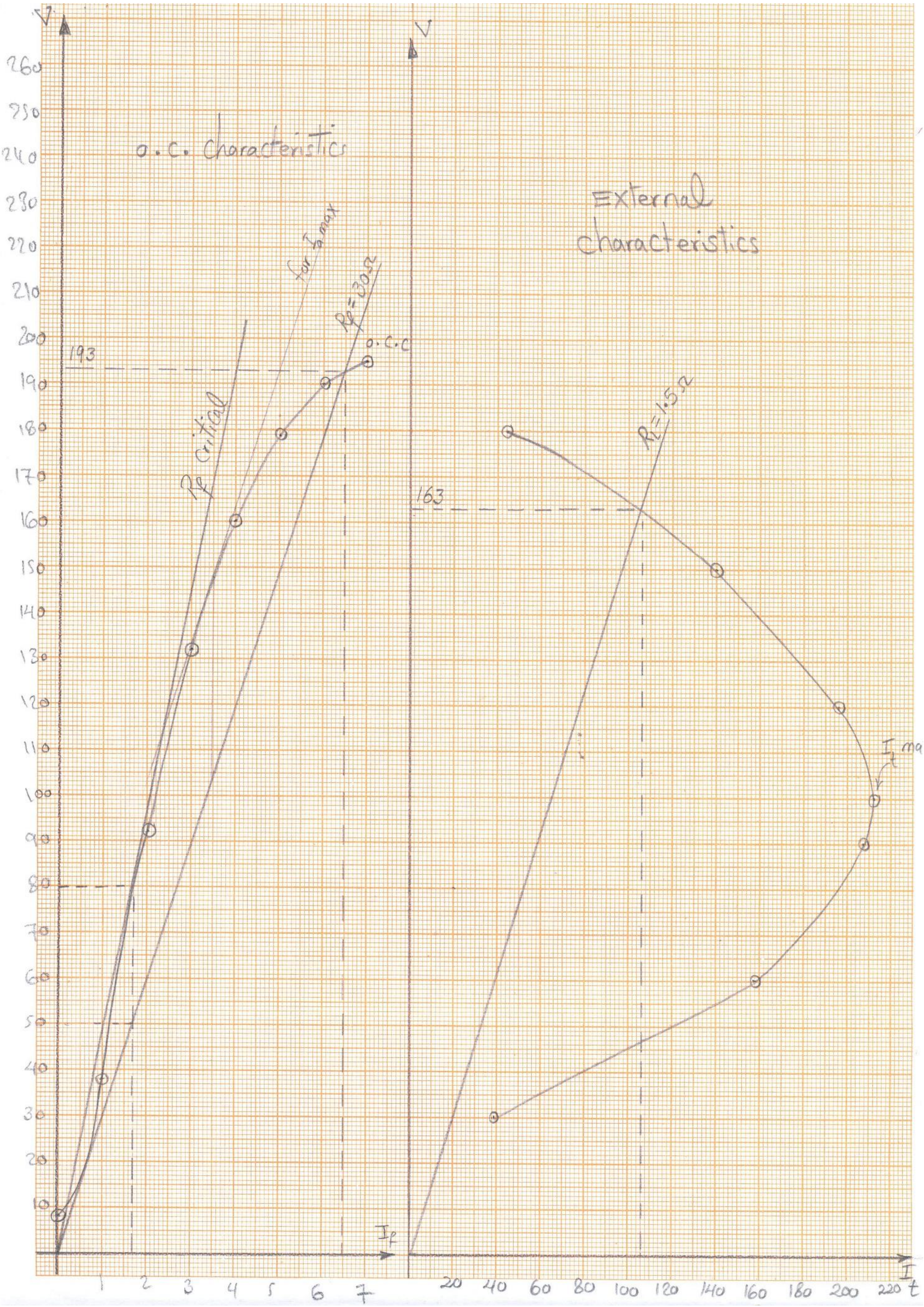
From external characteristics, draw a straight line I_tR_L (R_L=1.5Ω) that intersect at point C. it is found that Load voltage = 163 V, Load current = 106 A

The critical resistance is obtained by drawing the line I_fR_f that tangent OCC at point D. at that point V=80 V, I_f = 1.7 A

$$R_f critical = \frac{80}{1.7} = 47.06\ \Omega$$

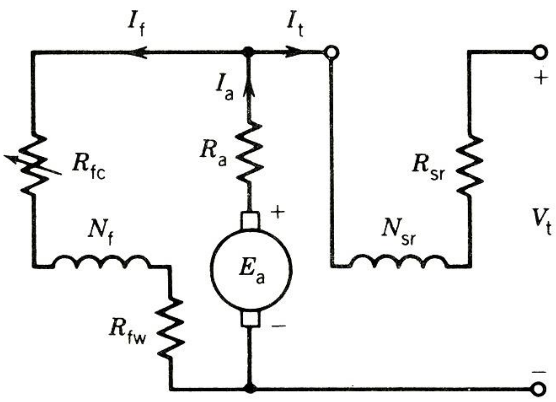

V
O.C.C.
Field Current (I_f)

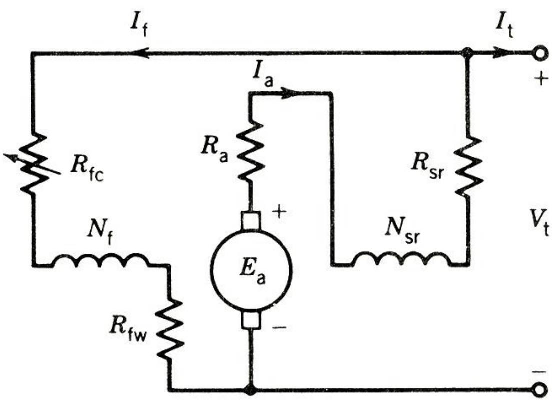

V
External Characteristic
Terminal Current (I_t)


o.c. characteristics
External characteristics
for I_f max
$R_f = 30\Omega$
R_f critical
o.c.c
193
163
$R_L = 1.5\Omega$
I_t max
V
I_f
I_t
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
1
2
3
4
5
6
7
20
40
60
80
100
120
140
160
180
200
220

3-Compound DC Generator

When DC machines deliver current, the terminal voltage drops because of I_aR_a voltage drop and a decrease in pole fluxes caused by armature reaction. To overcome the effects of I_aR_a drop, a winding can be mounted over the field poles along with the shunt field winding. This additional winding, known as a series winding, is connected in series with the armature winding and carries the armature current. This series winding may provide additional ampere-turns to increase or decrease pole fluxes, as desired. A DC machine that has both shunt and series windings is known as a compound machine. A schematic diagram of the compound machine is shown below.





Short Shunt

$$V_t = E_a - I_aR_a - I_tR_{sr}$$

$$I_t = I_a - I_f$$

Long Shunt

$$V_t = E_a - I_a(R_a + R_{sr})$$

$$I_t = I_a - I_f \;\; \& \;\; I_f = \frac{V_t}{R_{f\omega} + R_{fc}}$$

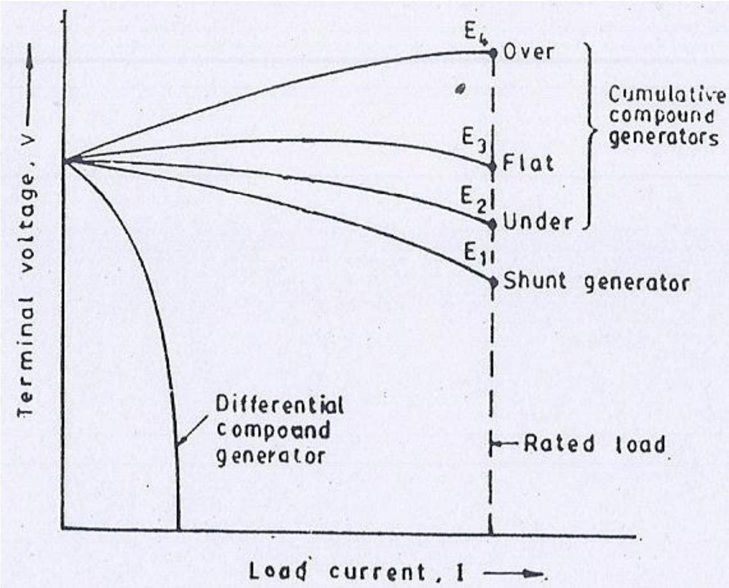
The shunt field winding is the main field winding, providing the major portion of the mmf in the machine. It has many turns of smaller cross-sectional area and carries a lower value of current compared to the armature current. The series winding has fewer turns, larger cross-sectional area, and carries the armature current. It provides mmf primarily to compensate the voltage drops caused by I_aR_a

For either connection, assuming magnetic linearity, the generated voltage is

$$E_a = K_a(\Phi_{sh} \pm \Phi_{sr})\omega_m$$

When the two fluxes aid each other, the machine is called *cumulative compound*, when the two fluxes oppose each other, the machine is called *differential compound*.

The voltage–current characteristics of the compound DC generator are shown in Fig. below. With increasing armature current the terminal voltage may rise (over compounding), decrease (under compounding), or remain essentially unchange (flat compounding). This depends on the degree of compounding—that is, the number of turns of the series field winding. For differential compounding (i.e., mmf of the series field winding opposed to that of the shunt field winding), the terminal voltage drops very quickly with increasing armature current. In fact, the armature current remains essentially constant. This current-limiting feature of the differentially compounded DC generator makes it useful as a welding generator.



Most commercial compound DC Machines, whether used as generators or motors, are normally supplied by the manufacturer as over-compound machines. The degree of compounding (over, flat or under) may be adjusted by means of divertor which shunts the series field.

The differential compound generator is used as a constant-current generator for the same constant-current applications as the series generator.

Calculation of Required Series Turns:

Consider a shunt generator that is converted to a short-compound generator by addition of a series field winding. From the test on the machine with shunt excitation only, it is found that a field current of 5 A gives 440 V on no-load and that 6 A gives 440 V at full load current of 200 A. The shunt winding has 1600 turns per pole. Find the number of series turns required.

It is seen that while running as a shunt generator, the increase in shunt field ampere-turns necessary for keeping its voltage constant from no-load to full-load is $N_{sh} \cdot \Delta I_{sh}$. This increase in field excitation can be achieved by adding a few series turns (N_s) to the shunt generator thereby converting it into a compound generator.

$$N_{sh} \times \Delta I_{sh} = N_s I_s$$

Where,

ΔI_{sh} is the increment in shunt field current required to keep voltage constant from no-load to full-load,

N_{sh} is the number of shunt field turns per pole,

N_s is the number of series turns per pole,

I_s is the current through series winding = armature current I_a for long-shunt and = load current I for short-shunt

It is given that for keeping the voltage of shunt generator constant at 440 V both at no-load and full-load, shunt field ampere-turns per pole have to be increased from 1600 × 5 = 8000 to (1600 × 6) = 9600 *i.e.* an increase of (9600 − 8000) = 1600 *AT*. This increase in field *AT* can be brought by adding a few series turns.

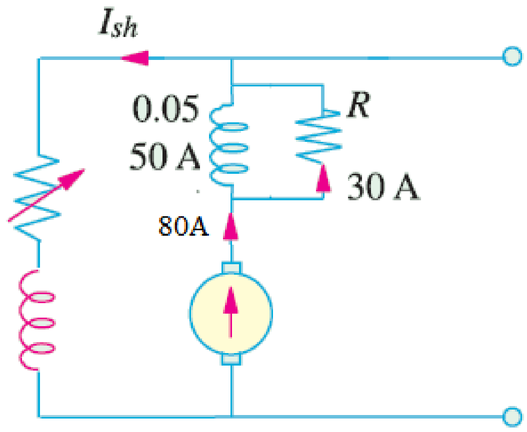
Let N_s be the number of series turns required per pole. Since they carry 200 A,

$$N_{sh} \times \Delta I_{sh} = N_s I_s$$

$$1600 \times 1 = N_s \times 200 \rightarrow N_s = 1600/200 = 8 \text{ Turns}$$

Example:

A long-compound generator has a shunt field winding of 1000 turns per pole and series field winding of 4 turns per pole and resistance 0.05 Ω. In order to obtain the rated voltage at no-load and full-load for operation as shunt generator, it is necessary to increase field current by 0.2 A. The full-load armature current of the compound generator is 80 A. Calculate the divertor resistance connected in parallel with series field to obtain flat compound operation.



Additional AT to maintain rated voltage at no-load and full-load = 1000 × 0.2 = 200

No. of series turns/pole = 4

Current required to produce 200 AT by the series field = 200/4 = 50 A.

Since I_a = 80 A, the balance of 30 A must pass through the parallel divertor resistance.

$\therefore$ 30 R = 50 × 0.05, $\quad\rightarrow\quad$ R = 0.0833 Ω

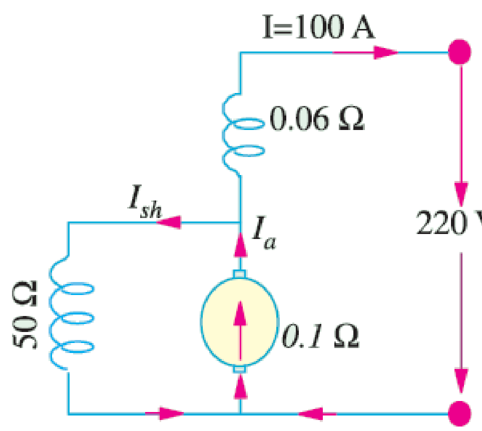
Example:

A 220-V compound generator is supplying a load of 100 A at 220 V. The resistances of its armature, shunt and series windings are 0.1 Ω, 50 Ω and 0.06 Ω respectively. Find the induced e.m.f. and the armature current when the machine is connected

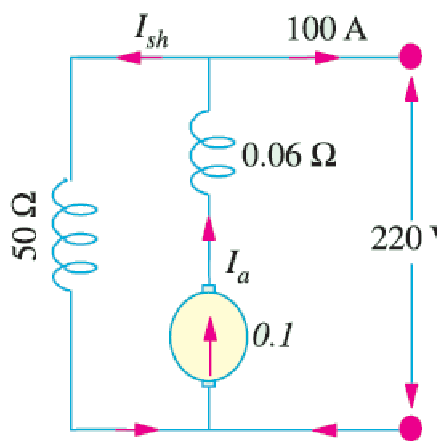
(a) short shunt **(b)** long shunt

(c) By what % will the series amp-turns be changed in **(b)** if a divertor of 0.14 Ω is connected in parallel with the series windings?

a) In case of short-compound connection:



Case a) Short compound



Case b) Long compound

$$I_{sh} = \frac{220 + 100 \times 0.06}{50} = 4.52\ A$$

$I_a = I_{sh} + I = 100 + 4.52 = 104.52$ A

$E_a = 220 + 104.52 \times 0.1 + 100 \times 0.06 = 236.452$ V

b) In case of long-compound connection:

$$I_{sh} = \frac{220}{50} = 4.4\ A$$

$I_a = I_{sh} + I = 100 + 4.4 = 104.4$ A

$$E_a = 220 + 104.4 \times (0.1 + 0.06) = 236.704\ V$$

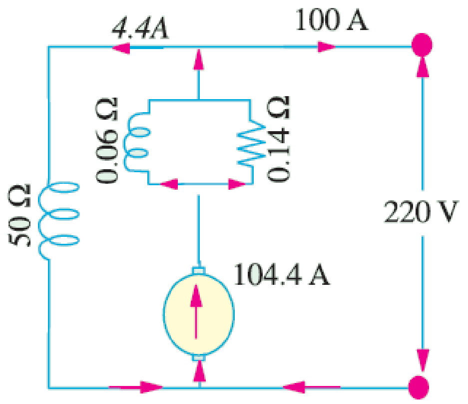
$$\text{Series AT} = 104.4\ N_s$$

c) After divertor resistance

$I_{divertor} = 104.4 \times 0.14/0.2 = 73.08$ A

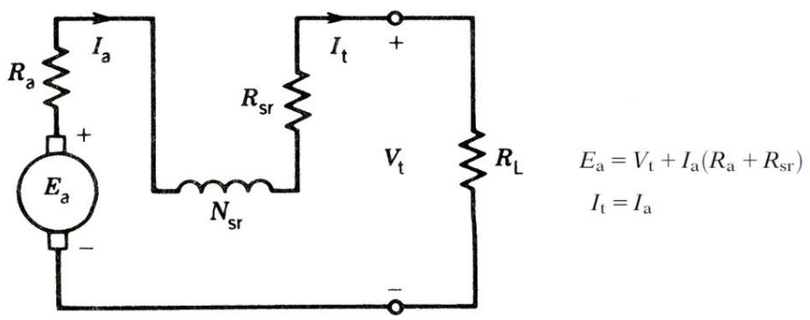
$$\text{Series AT} = 73.08\ N_s$$

%change in series AT = $(104.4\ N_s - 73.08\ N_s)/104.4\ N_s = 30\%$ (Down)



4-Series DC Generator

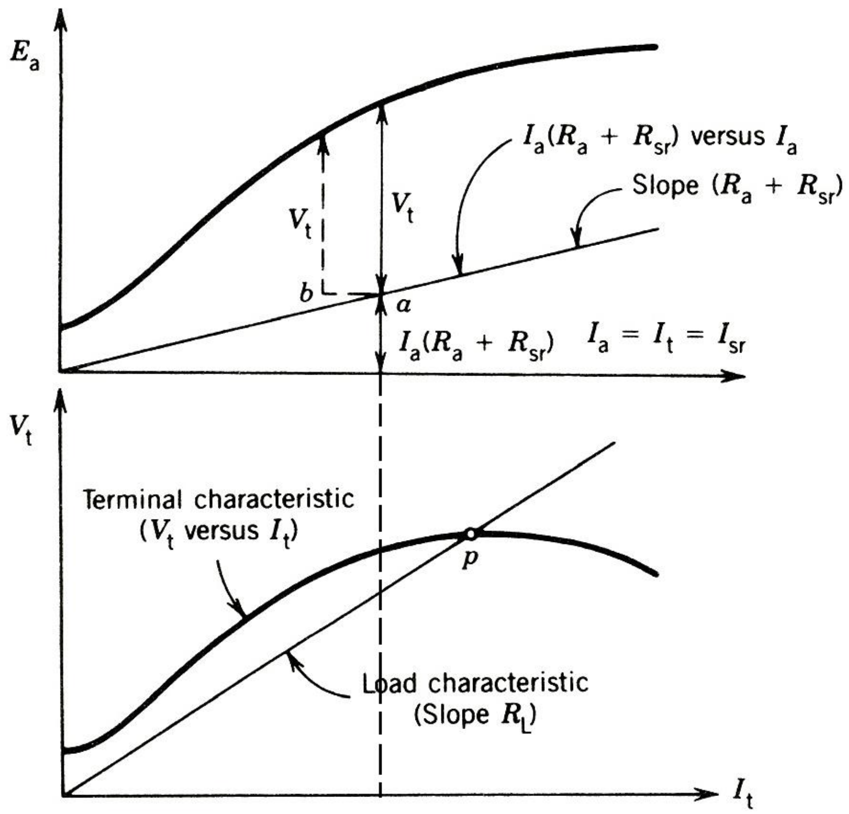
The circuit diagram of a series generator is shown below. The series field winding provides the flux in the machine when the armature current flows through it. The field circuit is not complete unless a load is connected to the machine.



The O.C.C (E_a versus I_a) for the series machine can be obtained by separately exciting the series field.

To obtain the external characteristic (i.e., V_t versus I_t), draw a straight line having the slope $R_a + R_{sr}$. This straight line represents the voltage drop across R_a and R_{sr}. The vertical distance between the O.C.C curve and this straight line is the terminal voltage for a particular value of I_a.

The terminal voltages for various values of the terminal current can thus be obtained as plotted in Fig. below. If R_L represent the load, the load characteristic, $V_t = R_L I_t$ versus I_t, is a straight line with slope R_L. The operating point for this load is the point of intersection of O.C.C curve with the load characteristic (point P). Note that if R_L is too large, the terminal voltage will be very small—that is, the series generator will not build up any appreciable voltage.



Example:

The OCC of a series generator at 1200 rpm is given in the following table:

Field Current (A)	10	20	30	40	60	80	100	120	140	160	170	180	190
Induced Voltage(V)	60	80	100	117	129	138	145	150	154	156	157	158	160

The armature resistance and series field resistance are 0.01 Ω and 0.2 Ω respectively. Fill the following table then sketch the external characteristic of that generator.

Field Current (A)	10	20	30	40	60	80	100	120	140	160	170	180	190
Induced Voltage (V)	60	80	100	117	129	138	145	150	154	156	157	158	160
$I_a(R_a+R_s)$													
$V_t = E_a - I_a(R_a+R_s)$													

a) If the load resistance is 1.0Ω, calculate the terminal voltage and current.

b) We need to adjust the terminal voltage to 110 V, what is the load resistance?

c) When the no-load voltage is 140 V, what is the power delivered?

To sketch the external characteristic of the series generator, the table will be:

Field Current (A)	10	20	30	40	60	80	100	120	140	160	170	180	190
Induced Voltage (V)	60	80	100	117	129	138	145	150	154	156	157	158	160
$I_a(R_a+R_s)$	2.1	4.2	6.3	8.4	12.6	16.8	21	25.2	29.4	33.6	35.7	37.8	39.9
$V_t = E_a - I_a(R_a+R_s)$	57.9	75.8	93.7	108.6	116.4	121.2	124	124.8	124.6	122.4	121.3	120.2	120.1

The external characteristic is shown below.

a) At $R_L = 1.0\ \Omega$, the load line (OA) intersects the external characteristic at

$$V_t = 125\text{ V} \quad \text{and } I_a = 125\text{ A}$$

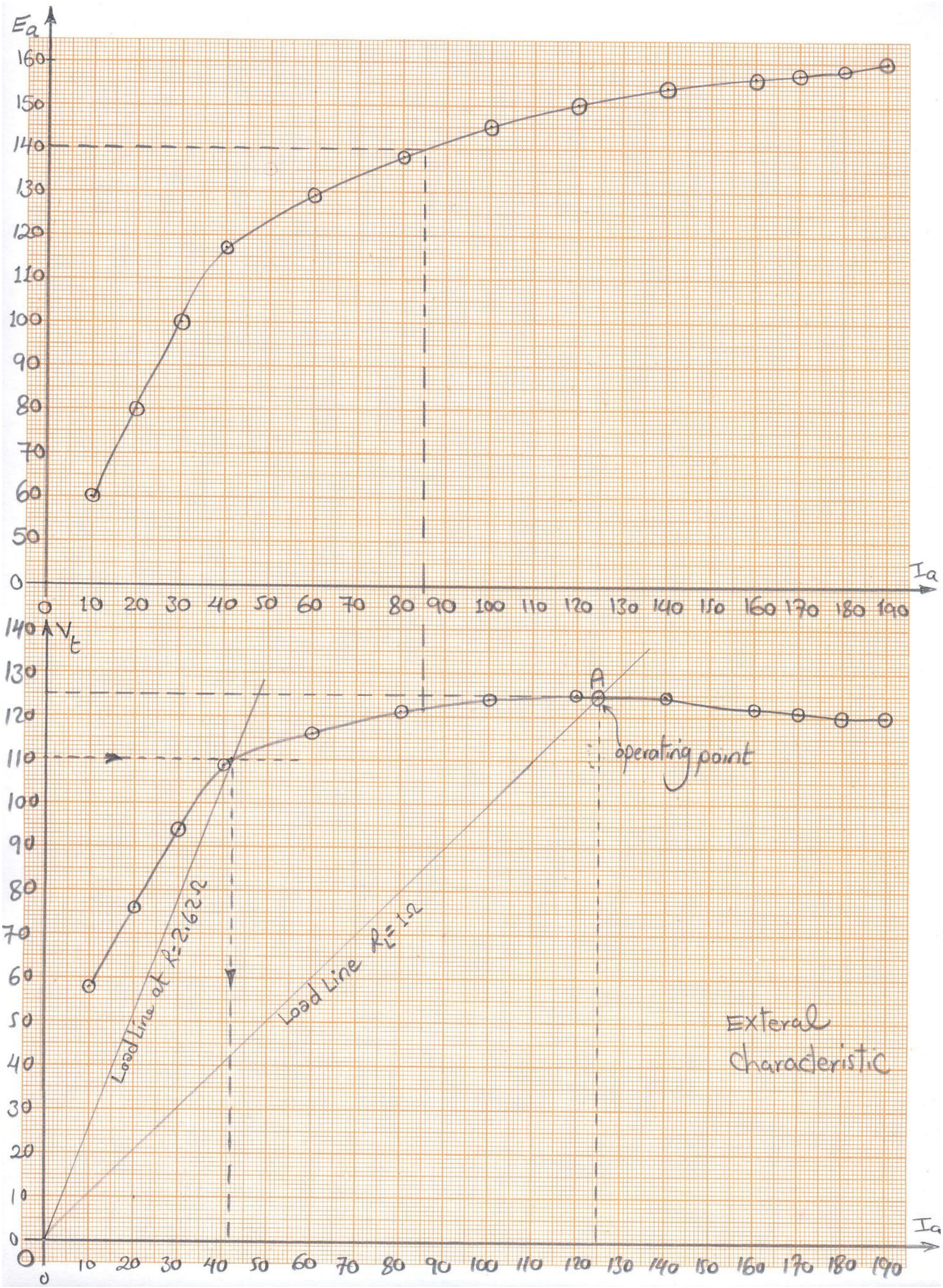
b) From the external characteristic, at $V_t = 110$ v, we find $I_a = 42$ A

Then $R_L = 110/42 = 2.62\ \Omega$

c) From OCC at $E_a = 140$ v, we find $I_a = 85$ A.

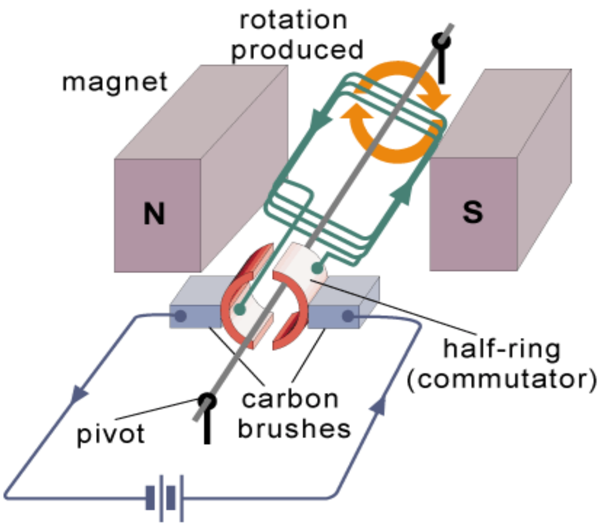
From external characteristic at $I_a = 85$ A, we find $V_t = 122.15$ v.

Then the power delivered $= 122.15 \times 85 = 10382.75$ W


Ea
Ia
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190
0 50 60 70 80 90 100 110 120 130 140 150 160
Vt
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140
A
operating point
Load Line at R=2.62 Ω
Load Line RL=1 Ω
Exteral
Characteristic

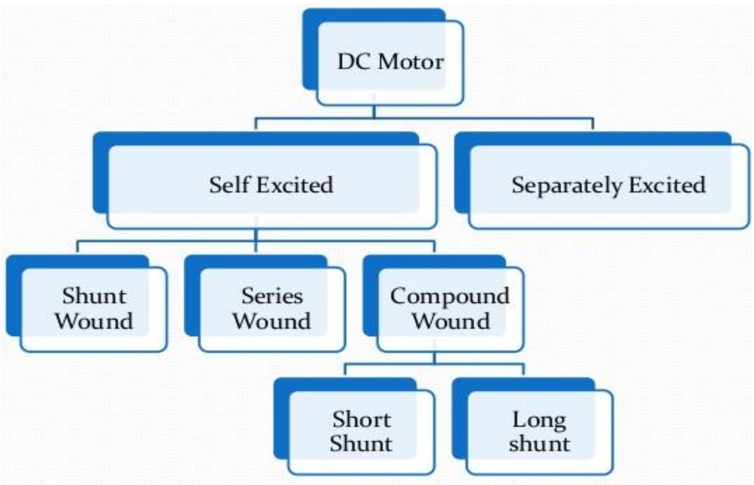
DC Motors

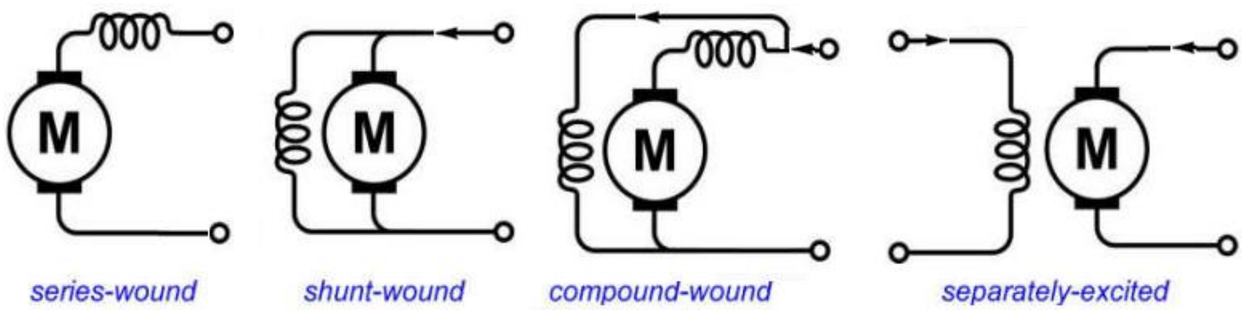
When the DC machine operates as a motor, the input to the machine is electrical power, and the output is mechanical power. If the armature is connected to a dc supply, the motor will develop mechanical torque and power.



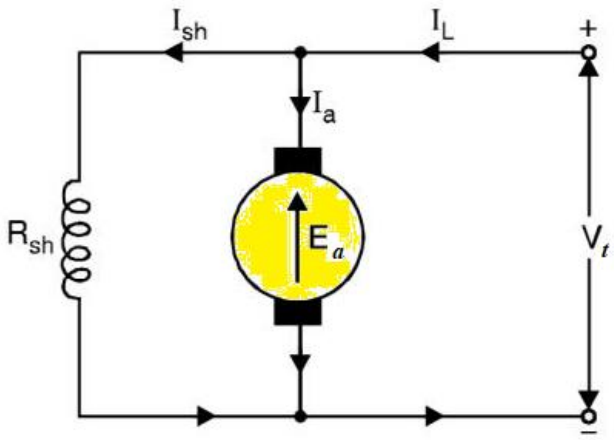
In fact, the DC machine is used more as a motor than as a generator. DC motors can provide a wide range of accurate speed and torque control.

Different types of DC motors





In case of DC motor,



$$V_t = E_a + I_a R_a$$

$$I_a = \frac{V_t - E_a}{R_a}$$

$$I_L = I_a + I_{sh}$$

Back e.m.f. (E_a) depends on the armature speed. If speed is high, E_a is large, but the armature current I_a is decreased as seen from the above equation. If the speed is less, then E_a is less, hence more current flows which develops motor torque. Therefore, at starting where the motor speed is zero, a huge armature current can flow which called starting current. There many techniques are used to limit such starting current. So, we find that E_a acts like a governor for self-regulating the armature current so that the motor draws as much current as is just necessary.

Example:

A 4-pole motor is fed at 440 V and takes an armature current of 50 A. The resistance of the armature circuit is 0·28 Ω. The armature winding is wave-connected with 888 conductors and useful flux per pole is 0·023 Wb. Calculate the speed of the motor.

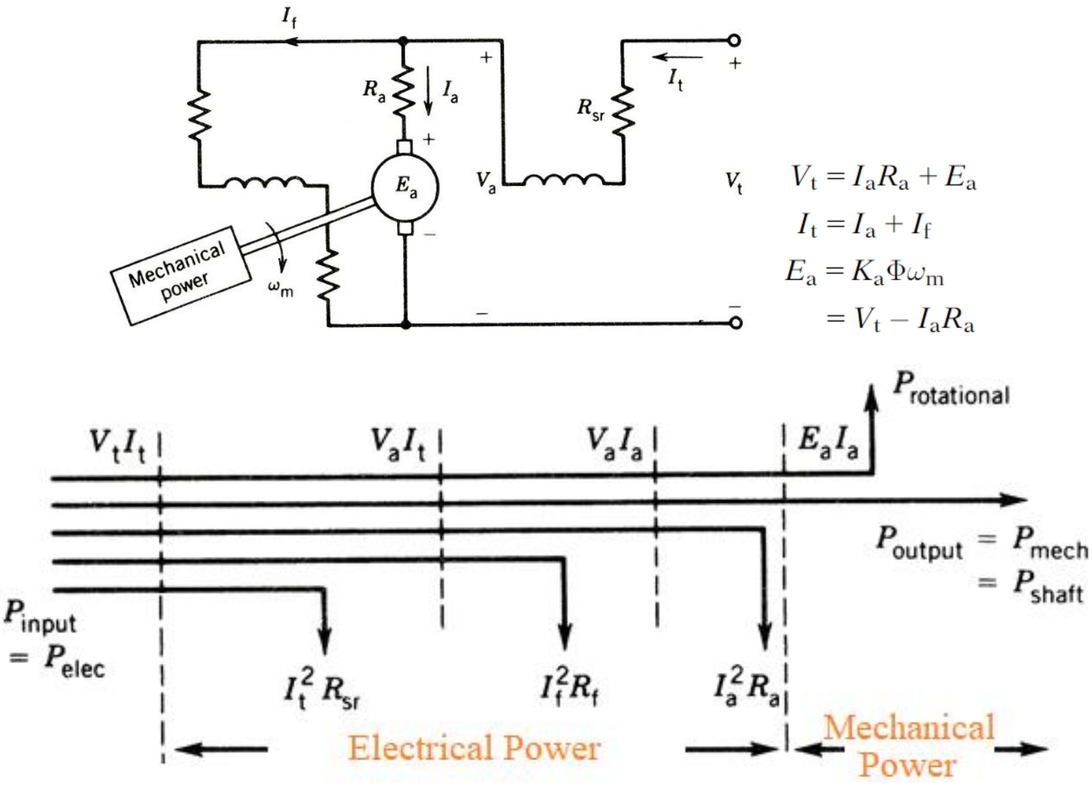
$$E_a = V_t - I_a R_a = 440 - 50 \times 0.28 = 426\ V$$

$$K_a = \frac{4 \times 888}{2\pi \times 2} = \frac{888}{\pi} = 282.6592$$

$$\omega = \frac{E_a}{K_a \times \emptyset} = \frac{426}{282.6592 \times 0.023} = 65.5268$$

$$N = \frac{60\ \omega}{2\pi} = \frac{60 \times 65.5268}{2\pi} = 625.7344$$

Power flow for DC motors



The electromechanical power (P_{em}) is the electrical power that just converted to mechanical power.

$$P_{em} = E_a I_a$$

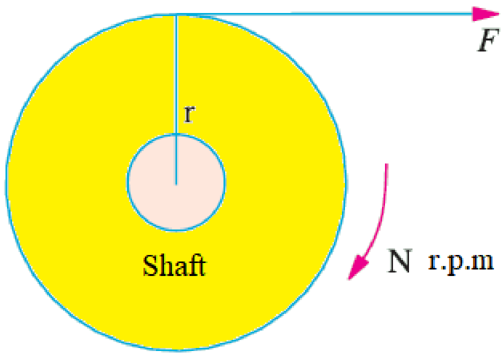
This electromechanical power (P_{em}) = $P_{rotational}$ + P_{shaft} = $P_{mech\ Gross}$

The Gross mechanical output power = $T_{Gross} \times \omega$ in Watts

$$\omega = \frac{2\pi\ N}{60}\ rad/s$$

Torque is the product of the force (F) in Newton and the radius (r) in meter at which this force acts as shown below.

$$T = F \times r \quad \text{(N.m)}$$



This gross mechanical torque includes the net torque (T_{net}) and the friction & windage or (rotational) losses (T_{Loss})

$$T_{Gross} = T_{shaft} + T_{Loss}$$

The net mechanical power (P_{shaft}) = $T_{shaft} \times \omega$ in Watts

The Rotational loss power ($P_{rotational}$) = $T_{Loss} \times \omega$ in Watts

Also, as said earlier

$$E_a I_a = T_{Gross} \times \omega$$

$$T_{Gross} = \frac{E_a I_a}{\omega} = \frac{K_a \emptyset\ \omega\ I_a}{\omega} = K_a \emptyset\ I_a$$

Where, K_a is the machine constant = $2P\ Z/(2\pi\ a)$

<u>Example</u>

A DC motor takes an armature current of 110 A at 480 V. The armature circuit resistance is 0.2 Ω. The machine has 6-poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05 Wb. Calculate **(i)**, the speed in r.p.m and **(ii)** the gross torque developed by the armature.

$$K_a = \frac{6 \times 864}{2\pi \times 6} = 137.5099$$

$E_a = V_t - I_a R_a = 480 - 110 \times 0.2 = 458$ V

But $E_a = K_a\ \Phi\ \omega \rightarrow \omega = E_a / (K_a \times \Phi)$

$$\omega = \frac{E_a}{K_a \emptyset} = \frac{458}{137.5099 \times 0.05} = 66.6134\ rad/s$$

$N = \omega \times 60/(2\pi) = 636.111$ r.p.m

$$E_a I_a = T_{Gross} \times \omega$$

$$T_{Gross} = \frac{E_a I_a}{\omega} = \frac{458 \times 110}{66.6134} = 756.3043\ N$$

Example:

The armature winding of a 4-pole, 250 V DC shunt motor is lap connected. There are 120 slots, each slot containing 8 conductors. The flux per pole is 20 mWb and current taken by the motor is 25 A. The resistance of armature and field circuit are 0.1 and 125 Ω respectively. If the rotational losses amount to be 810 W find,

- **(v)** gross torque
- **(vi)** useful torque and
- **(vii)** efficiency.

$I_{sh} = 250/125 = 2$ A

$I_a = 25 - 2 = 23$ A

$E_b = 250 - (23 \times 0.1) = 247.7$ V

Z = 120 × 8 = 960

$$K_a = \frac{4 \times 960}{2\pi \times 4} = 152.7887$$

Gross torque (T_{Gross}) = $K_a\ \Phi\ I_a$ = 152.7887 × 0.02 × 23 = 70.2828 N.m

Another method

$$\omega = \frac{E_a}{K_a \emptyset} = \frac{247.7}{152.7887 \times 0.02} = 81.0597\ rad/s$$

$$T_{Gross} = \frac{E_a I_a}{\omega} = \frac{247.7 \times 23}{81.0597} = 70.2828\ N.m$$

$P_{out} = P_{em} - P_{rotational}$ = 5697.1 – 810 = 4887.1 W

Useful Torque = P_{out} / ω = 4887.1 / 81.0597 = 60.2901 N.m

P_{in} = V × I = 250 × 25 = 6250 W

η = P_{out} / P_{in} = 4887.1 / 6250 = 78.1936 % #

Example

A 500-V, 37.3 kW, 1000 r.p.m. DC shunt motor has on full-load an efficiency of 90%. The armature circuit resistance is 0.24 Ω and there is total voltage drop of 2 V at the brushes. The field current is 1.8 A. Determine

(i) Full-load line current

(ii) Full load shaft torque in N.m

(iii) Rotational loss in W

(iv) Total resistance in motor starter to limit the starting current to 1.5 times the full-load current.

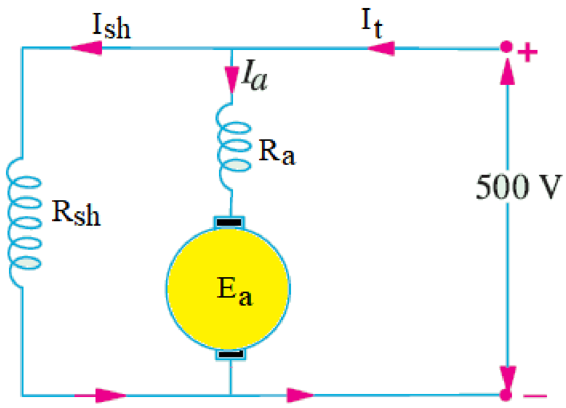
P_{out} = 37300 W

$P_{in} = P_{out}/\eta$ = 37300/0.9 = 41444.4444 W

$P_{in} = V_t\, I_t \quad \rightarrow \quad I_t = P_{in}/V_t$ = 41444.4444/500 = 82.8889 A ##

$\omega = 2\times\pi\times1000/60$ = 104.7198 rad/s

$P_{out} = T_{shaft} \times\omega \quad \rightarrow \quad T_{shaft} = P_{out}/\omega$ = 37300/104.7198 = 356.1888 N.m ##



$I_a = I_t - I_{sh}$ = 82.8889 – 1.8 = 81.0889 A

$E_a = V_t - I_aR_a - V_b$ = 500 - 81.0889×0.24 – 2 = 478.5387 V.

$P_{em} = E_a\, I_a$ = 478.5387×81.0889 = 38804.1768 W

$P_{Rotational} = P_{em} - P_{out}$ = 38804.1768 – 37300 = 1504.1768 W ##

At starting $\omega = 0 \rightarrow E_a = 0$

Also given that the line current is limited to 1.5 times

I_t = 1.5×82.8889 = 124.3333 A

I_a = 124.3333 – 1.8 = 122.5333 A

$V_t = I_a\,(R_a + R_{starter}) + V_b \quad \rightarrow \quad 500 = 122.5333(0.24+R_{starter}) + 2$

$R_{starter} = 3.8242\ \Omega$ ##

Example

A 4-pole, 240 V, wave connected shunt motor gives 11.19 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 60 slots with 9 conductors per slot. Its resistance is 0.1 Ω. Assuming a drop of 1 volt per brush, find

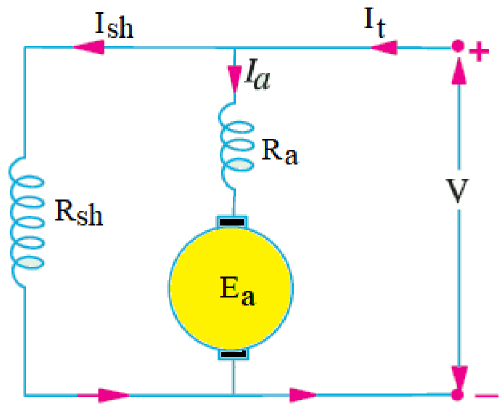
(a) Gross (total) torque

(b) Shaft (useful) torque

(c) useful flux / pole

(d) rotational losses

(e) efficiency.



$I_t = I_a + I_{sh} = 50 + 1 = 51$ A

$P_{in} = V_t \times I_t = 240\times51 = 12240$ W

$E_a = V_t - I_aR_a - V_b = 240 - 50\times0.1 - 2 = 233$ V.

$P_{em} = E_a\, I_a = 233\times50 = 11650$ W

$P_{Rotational} = P_{em} - P_{out} = 11650 - 11190 = 460$ W ##

$\omega = 2\pi\times1000/60 = 104.7198$ rad/s.

$P_{out} = T_{Shaft} \times \omega \quad \rightarrow \quad T_{Shaft} = P_{out} / \omega = 11190/104.7198 = 106.8566$ N.m #

$P_{mech\ Gross} = P_{em} = T_{Gross} \times \omega \quad \rightarrow \quad T_{Gross} = P_{em} / \omega = 11650/104.7198 = 111.2493$ N.m #

$Z = 60 \times 9 = 540$ conductors

$K_a = 2P\, Z/(2\pi\, a) = 4\times540/(2\pi\times2) = 171.88734$

$E_a = K_a \times \Phi \times \omega \quad \rightarrow \quad \Phi = E_a / (K_a \times \omega) = 233/(171.88734\times104.7198)=12.944$ mWb #

$\eta = P_{out} / P_{in} = 11190 / 12240 = 91.4216$ % #

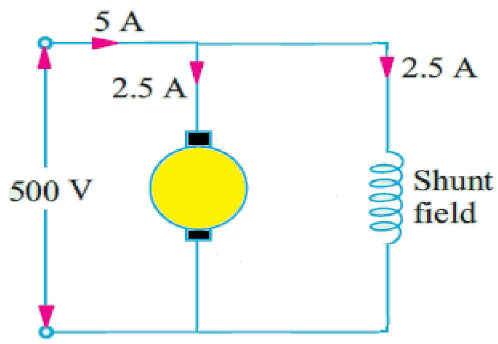
Example

A 500-V DC shunt motor draws a line-current of 5 A on No-load and running at 600 rpm. If the armature resistance is 0.15 Ω and the field resistance is 200 Ω, determine

a) Efficiency of the machine running as a generator delivering a load current of 40 A.

b) At what speed should the generator be run, if the shunt-field is not changed

When the machine runs as a motor:



E_{am} = 500 – 2.5×0.15 = 499.625 V,

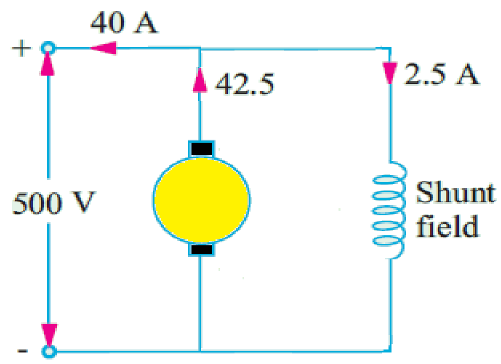
Input power = 500×5 = 2500 W,

Shunt-field Cu loss = 2.5^2 × 200 = 1250 W,

Armature Cu loss = 2.5^2 × 0.15 = 0.9375 W,

Fixed loss (Core + rotational) losses = 2500 – 1250 – 0.9375 = 1249.0625 W,

When the machine runs as a generator:



E_{ag} = 500 + 42.5×0.15 = 506.375 V

Output power = 500×40 = 20000 W,

Shunt-field Cu loss = 2.5^2 × 200 = 1250 W,

Armature Cu loss = 42.5^2 × 0.15 = 270.9375 W,

Fixed loss still as in motor case (same machine) 1249.0625 W,

$$\eta = \frac{P_{out}}{P_{out} + P_{Losses}} = \frac{20000}{20000 + 1250 + 270.9375 + 1249.0625} = 87.8349\%$$

$$\frac{E_{ag}}{E_{am}} = \frac{N_g}{N_m}$$

$$\frac{506.375}{499.625} = \frac{N_g}{600} \rightarrow N_g = 608.11\, rpm$$

Example

A 460-V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and % change in torque if the load is reduced so that the motor is taking 30 A. Giving that total resistance of the armature and field circuits is 0.8 Ω.

In case of Load#1 that takes 40A:

$E_{a1} = 460 - 40\times0.8 = 428$ V

$E_{a1} = K_a\, \Phi\, \omega$ but Φ is directly proportional to I_a $\rightarrow$ $E_{a1} = K_a\, I_a\, N_1$

$E_{a1} = K_a\, (40)\, N_1$

$T_1 = K_a\, \Phi\, I_a$ but Φ is directly proportional to I_a $\rightarrow$ $T_1 = K_a\, I_a^2$

$T_1 = K_a\, (40)^2$

In case of Load#2 that takes 30A:

$E_{a2} = 460 - 30\times0.8 = 436$ V

$E_{a2} = K_a\, \Phi\, \omega$ but Φ is directly proportional to I_a $\rightarrow$ $E_{a2} = K_a\, I_a\, N_1$

$E_{a2} = K_a\, (30)\, N_2$

$T_2 = K_a\, \Phi\, I_a$ but Φ is directly proportional to I_a

$T_2 = K_a\, I_a^2$ $T_2 = K_a\, (30)^2$

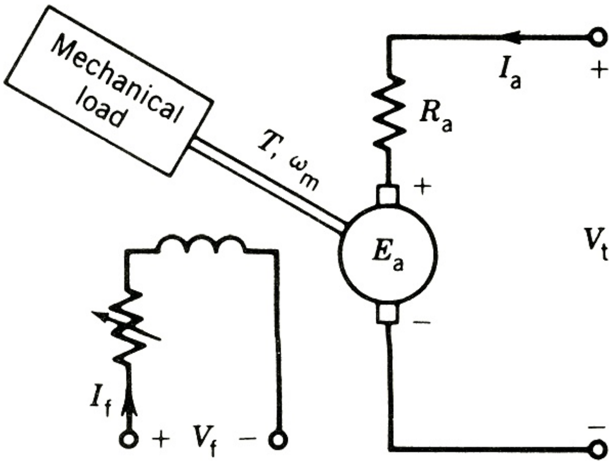
$$\frac{E_{a1}}{E_{a2}} = \frac{I_{a1}N_1}{I_{a2}N_2} \rightarrow \frac{428}{436} = \frac{40 \times 500}{30 \times N_2} \rightarrow N_2 = 679.1277\, rpm$$

$$\frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2 \rightarrow \frac{T_1}{T_2} = \left(\frac{40}{30}\right)^2 = \frac{16}{9}$$

Torque-Speed Characteristics (External Characteristics)

In many applications DC motors are used to drive mechanical loads. Some applications require that the speed remain constant as the mechanical load applied to the motor changes.

Consider the separately excited DC motor shown below. The voltage, current, speed, and torque are related as also given.



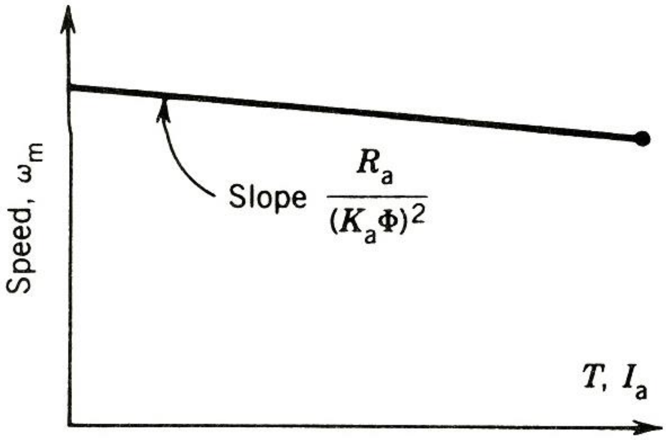
$$E_a = K_a \Phi \omega_m = V_t - I_a R_a$$

$$T = K_a \Phi I_a$$

$$\omega_m = \frac{V_t - I_a R_a}{K_a \Phi}$$

$$\omega_m = \frac{V_t}{K_a \Phi} - \frac{R_a}{(K_a \Phi)^2} T$$

If the terminal voltage V_t and machine flux Φ are kept constant, the torque–speed characteristic is as shown below. The drop in speed as the applied torque increases is small, providing a good speed regulation. In an actual machine, the flux Φ will decrease because of armature reaction as T or I_a increases, and as a result the speed drop will be less than that shown. The armature reaction therefore improves the speed regulation in a DC motor.



Speed equation given above suggests that speed control of DC motor can be achieved by the following methods:

1. Armature voltage control (V_t).
2. Field control (Φ).
3. Armature resistance control (R_a).

In fact, speed in a DC motor increases as V_t increases and decreases as Φ or R_a increases.

Speed Control by Changing Armature Voltage

Assuming that the field current is kept constant (Φ is constant) and assuming the armature resistance is unchanged (R_a = fixed value), then the relation between the motor speed and terminal voltage (V_t) is;

$$\omega_m = \frac{V_t}{K_a\Phi} - \frac{R_a}{(K_a\Phi)^2}T$$

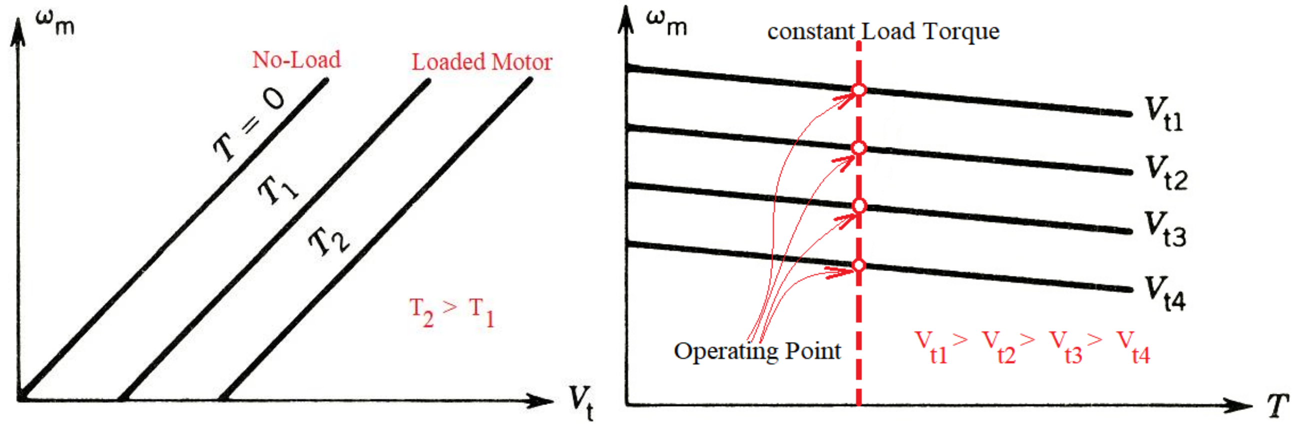
$$\omega_m = K_1 V_t - K_2 T$$

where $K_1 = 1/K_a\Phi$

$K_2 = R_a/(K_a\Phi)^2$

There are two types of load torque; constant torque (such as that applied by an elevator or hoist crane) and variable torque (such as that applied by a fan loads)

Therefore, for a constant load torque, the speed will change linearly with V_t as shown in figures below.



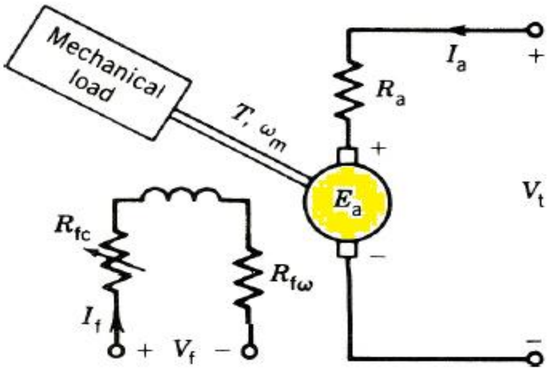
In an actual application, when speed is changed by changing the terminal voltage, the armature current is kept constant (needs a closed-loop operation).

As load torque is constant, then I_a is also constant and the power is changed linearly with speed as V_t changing from $0 \rightarrow V_{base}$.

The armature voltage control scheme provides a smooth variation of speed control from zero to the base speed. The base speed is defined as the speed obtained at rated terminal voltage. This method of speed control is, however, expensive, because it requires a variable DC supply for the armature circuit.

<u>Speed Control by Changing Field current</u>

In this method the armature circuit resistance R_a and the terminal voltage V_t remain fixed and the speed is controlled by varying the current (I_f) of the field circuit. This is normally achieved by changing the field circuit rheostat (R_{fc}) as shown below.

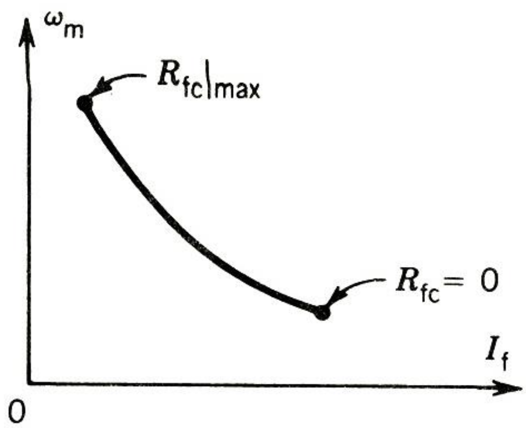


Assuming the magnetic flux is changed linearly with the field current, therefore the flux in the machine (Φ) will be proportional to the field current (I_f). $\Phi \;\alpha\; I_f$

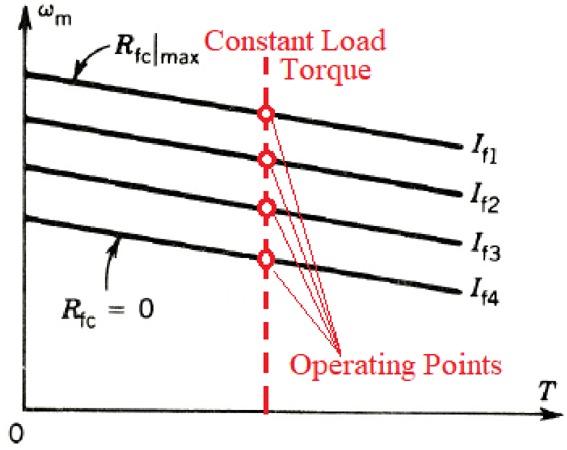
$$K_a\Phi = K_f I_f$$

$$\omega_m = \frac{V_t}{K_f I_f} - \frac{R_a}{(K_f I_f)^2}T$$

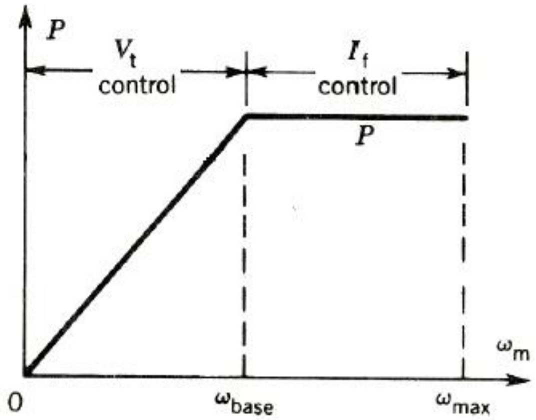
As the field circuit rheostat (R_{fc}) changed from R_{fc} (max) to R_{fc} (min), the speed is changed nonlinearly as shown below.

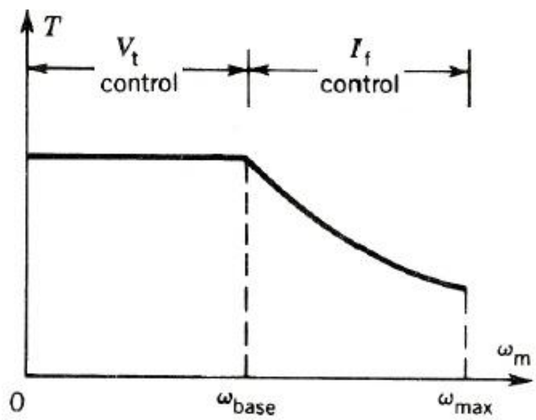


As the field resistance increases, the field current decreases and the machine flux decreases. Therefore, the motor speed increases as shown in figure below.



Speed control from zero to a base speed is usually obtained by armature voltage control (V_t). Speed control beyond the base speed is obtained by decreasing the field current, called field weakening. At the base speed, the armature terminal voltage is at its rated value. If armature current is not to exceed its rated value (heating limit), speed control beyond the base speed is restricted to constant power, known as constant-power operation. The torque, therefore, decreases with speed in the field weakening region. The features of armature voltage control (constant-torque operation) and field control (constant-power operation) are shown in figures below.





In the armature voltage control mode, the motor current is kept constant at its rated value, and the motor terminal voltage V_t is changed from zero to its rated value. The speed will change from zero to the base speed. The torque can be maintained constant during operation in this range of speed, as shown above.

In field current control mode, the armature voltage V_t remains constant and the motor field current is decreased (field weakening) to obtain higher speeds. The armature current can be kept constant, thereby operating the motor in a constant-horsepower mode. The torque obviously decreases as speed increases, as shown above.

Example:

A variable-speed drive system uses a dc motor that is supplied from a variable-voltage source. The drive speed is varied from 0 to 1500 rpm (base speed) by varying the terminal voltage from 0 to 500 V with the field current maintained constant.

a) Determine the motor armature current if the torque is held constant at 300 N.m up to the base speed.
b) The speed beyond the base speed is obtained by field weakening while the armature voltage is held constant at 500 V. Determine the torque available at a speed of 3000 rpm if the armature current is held constant at the value obtained in part (a). Neglect all losses.

(a) $N_b = 1500 \text{ rpm}, \qquad V_t = 500 \text{ V} \simeq E_a$

$$K_a\Phi = \frac{500}{1500 \times 2\pi/60} = 3.1831$$

$$I_a = \frac{T}{K_a\Phi} = \frac{300}{3.1831} = 94.2477 \text{ A}$$

(b) $n = 3000 \text{ rpm}, \qquad V_t = E_a = 500 \text{ V}$

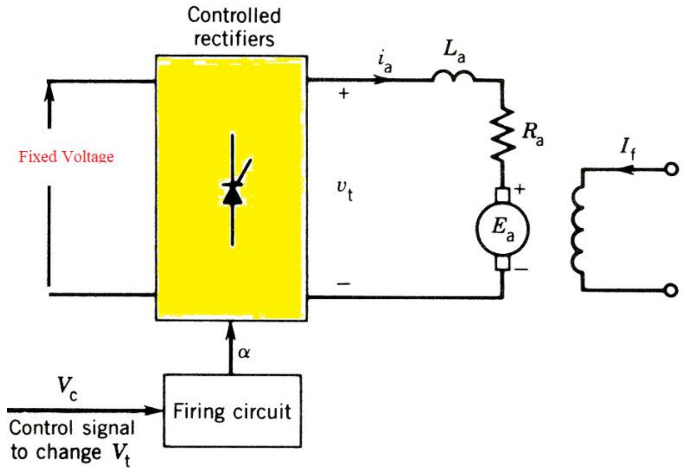
$$K_a\Phi = \frac{500}{3000 \times 2\pi/60} = 1.5916$$

$$T = 1.5916 \times 94.2477 = 150 \text{ N} \cdot \text{m}$$

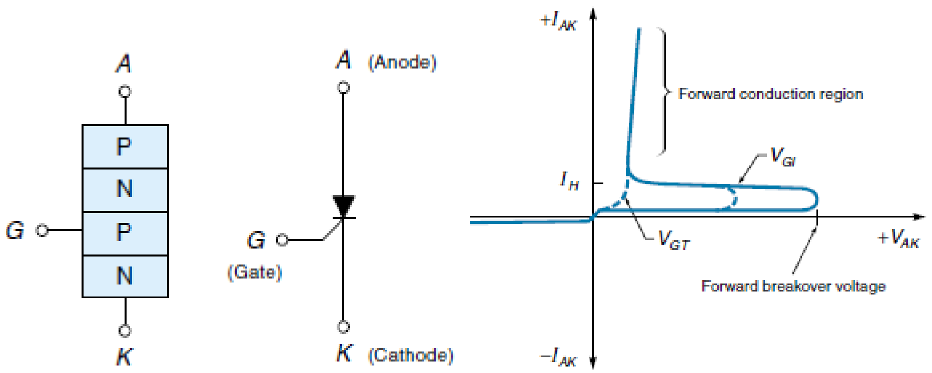
or $$T = \frac{P}{\omega_m} = \frac{500 \times 94.2477}{3000 \times 2\pi/60} = 150 \text{ N} \cdot \text{m}$$

Speed Control by Solid state

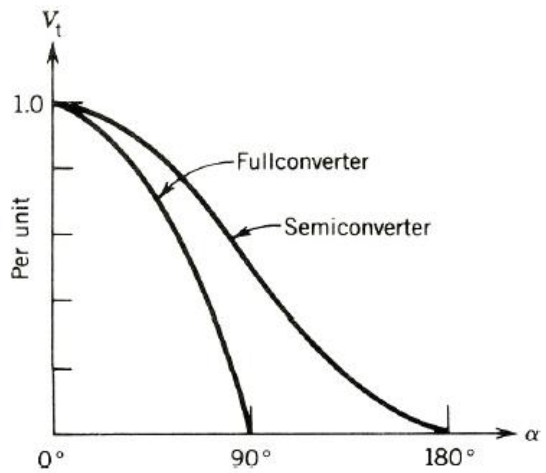
In recent years, solid-state converters have been used to produce a variable DC voltage in order to control the speed of DC motors as shown in figure below.



If the supply is AC (single-phase or three-phase), controlled rectifiers can be used to convert a fixed AC supply voltage to a variable-voltage DC supply. The main switching device of the Silicon Controlled Rectifier (SCR) is the thyristor shown below.



The converter is called a full converter, if the all elements of SCR are thyristors. The converter is called semiconverter, if some of the SCR elements are diodes. The relation between V_t and the firing angle (α) for a single-phase AC source for both types of converters is given below.



Series Motor

The circuit diagram of a series motor is shown in figure below. An external resistance R_{ae} is connected in series with the armature. This resistance can be used to control the speed of the series motor. Assuming magnetic linearity;

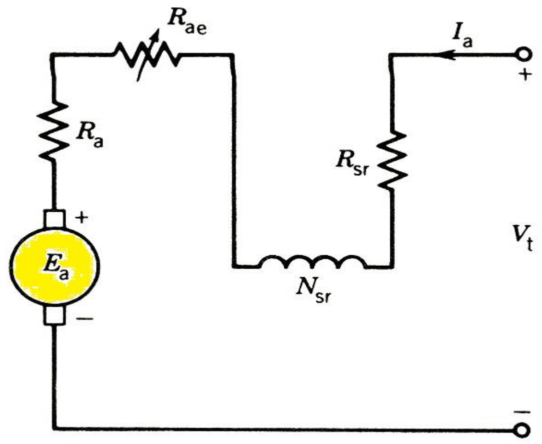
$$K_a\Phi = K_{sr}I_a$$

$$E_a = K_{sr}I_a\omega_m \quad \& \quad T = K_{sr}I_a^2$$

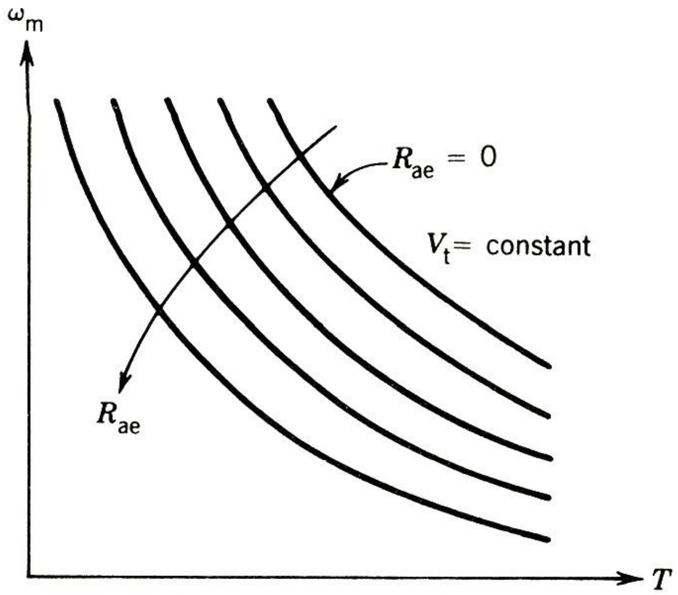
$$E_a = V_t - I_a(R_a + R_{ae} + R_{sr})$$

$$\omega_m = \frac{V_t}{K_{sr}I_a} - \frac{R_a + R_{sr} + R_{ae}}{K_{sr}}$$

$$\omega_m = \frac{V_t}{\sqrt{K_{sr}}\sqrt{T}} - \frac{R_a + R_{sr} + R_{ae}}{K_{sr}}$$



The torque–speed characteristics for various values of R_{ae} are shown below. For a particular value of R_{ae}, the speed is almost inversely proportional to the square root of the torque. A high torque is obtained at low speed and a low torque is obtained at high speed. Series motors are therefore used where large starting torques are required, as in subway cars, automobile starters, hoists, cranes, and blenders.



Example:

A 220 V, 7 hp series motor is mechanically coupled to a fan and draws 25 amps and runs at 300 rpm when connected to a 220 V supply with no external resistance connected to the armature circuit (i.e., R_{ae} =0). The torque required by the fan is proportional to the square of the speed. R_a = 0.6 Ω and R_{sr} = 0.4 Ω. Neglect armature reaction and rotational loss.

(a) Determine the power delivered to the fan and the torque developed by the machine.

(b) The speed is to be reduced to 200 rpm by inserting a resistance R_{ae} in the armature circuit. Determine the value of this resistance and the power delivered to the fan.

$$E_a = V_t - I_a(R_a + R_{sr} + R_{ae})$$
$$= 220 - 25(0.6 + 0.4 + 0) = 195 \text{ V}$$

$$P = E_a I_a = 195 \times 25 = 4880 \text{ W}$$

$$P = \frac{4880}{746} \text{ hp} = 6.54 \text{ hp}$$

$$T = \frac{E_a I_a}{\omega_m} = \frac{4880}{300 \times 2\pi/60} = 155.2 \text{ N} \cdot \text{m}$$

(b) $$T = K_{sr} I_a^2$$

$$155.2 = K_{sr} 25^2 \qquad K_{sr} = 0.248$$

$$T|_{200 \text{ rpm}} = \left(\frac{200}{300}\right)^2 \times 155.2 = 68.98 \text{ N} \cdot \text{m}$$

$$\frac{200}{60} \times 2\pi = \frac{200}{\sqrt{0.248}\sqrt{68.98}} - \frac{0.6 + 0.4 + R_{ae}}{0.248} \quad \rightarrow \quad R_{ae} = 7 \ \Omega$$

$$P = T\omega_m = 68.98 \times \frac{200}{60} \times 2\pi = 1444\text{W} \qquad \rightarrow 1.94 \text{ hp}$$

$$68.98 = 0.248 I_a^2$$

$$I_a = 16.68 \text{ amps}$$

$$E_a = K_{sr} I_a \omega_m = 0.248 \times 16.68 \times \frac{200}{60} \times 2\pi = 86.57 \text{ V}$$

$$E_a = V_t - I_a(R_a + R_{sr} + R_{ae})$$

$$86.57 = 220 - 16.68(0.6 + 0.4 + R_{ae}) \quad \rightarrow \quad R_{ae} = 7 \ \Omega$$

$$P = E_a I_a = 86.57 \times 16.68 = 1444 \text{ W} \quad \rightarrow 1.94 \text{ hp}$$

Example:

The following results were obtained from a static torque test on a DC series motor:

Current (A)	20	30	40	50
Torque (N.m)	128.8	230.5	349.8	469.2

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V. Resistance of armature and field winding is 0.5 Ω. Ignore iron and friction losses.

At $I_a = 20A \rightarrow T = 128.8$ N.m (first point in the table)

$P_{in} = V_t \times I_a = 460\times20 = 9200$ W

$P_{cu} = I_a^2 \times (R_a+R_s) = 400\times0.5 = 200$ W

$P_{out} = 9200 - 200 = 9000 \text{ W} = T \times \omega \rightarrow \omega = 9000/128.8 = 69.876$ rad/s

At $I_a = 30A \rightarrow T = 230.5$ N.m (second point in the table)

$P_{in} = V_t \times I_a = 460\times30 = 13800$ W

$P_{cu} = I_a^2 \times (R_a+R_s) = 900\times0.5 = 450$ W

$P_{out} = 13800 - 450 = 13350 \text{ W} = T \times \omega \rightarrow \omega = 13350/230.5 = 57.9176$ rad/s

At $I_a = 40A \rightarrow T = 349.8$ N.m (third point in the table)

$P_{in} = V_t \times I_a = 460\times40 = 18400$ W

$P_{cu} = I_a^2 \times (R_a+R_s) = 1600\times0.5 = 800$ W

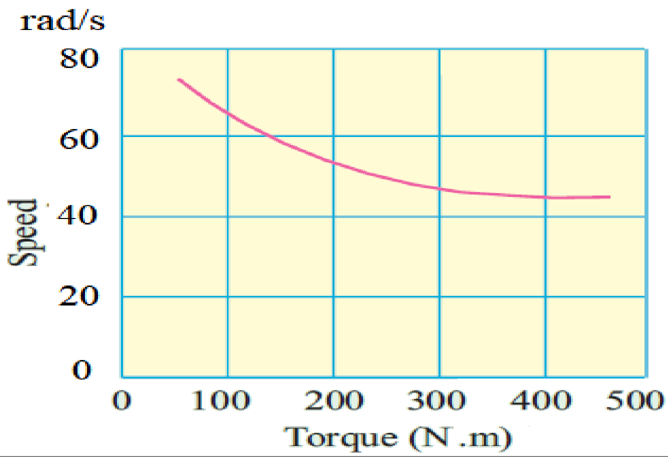
$P_{out} = 18400 - 800 = 17600 \text{ W} = T \times \omega \rightarrow \omega = 17600/349.8 = 50.3145$ rad/s

At $I_a = 50A \rightarrow T = 469.2$ N.m (fourth point in the table)

$P_{in} = V_t \times I_a = 460\times50 = 23000$ W

$P_{cu} = I_a^2 \times (R_a+R_s) = 2500\times0.5 = 1250$ W

$P_{out} = 23000 - 1250 = 21750 \text{ W} = T \times \omega \rightarrow \omega = 21750/469.2 = 46.3555$ rad/s



Example:

A fan which requires 8 HP at 700 rpm is coupled directly to a DC series motor. Calculate the input to the motor when the supply voltage is 500 V, assuming that power required for fan varies as the square of the speed. For the purpose of obtaining the magnetisation characteristics, the motor was running as a self-excited generator at 600 r.p.m. and the relationship between the terminal voltage and the load current was found to be as:

Current (A)	7	10.5	14	27.5
No-Load Voltage (V)	347	393	434	458

The resistance of both the armature and field windings of the motor is 3.5 Ω and the core, friction and other losses may be assumed to be constant at 450 W for the speeds corresponding to the above range of currents at normal voltage.

*At first point: I_a = 7 A & E_a = 347 at 600 rpm

With load at the same current; E_a = 500 - 7×3.5 = 475.5 V

$$\frac{347}{475.5} = \frac{600}{N} \rightarrow N = 822.19\, rpm$$

$P_{em} = E_a I_a$ = 475.5×7 = 3328.5 W

P_{out} = 3328.5 – 450 = 2878.5 W

The power required by the fan load $P_{fan} = K \times N^2$

We have one point at the load which is P_{fan} = 8×847 = 5968 W at 700 rpm

5968 =K×700^2 → K = 0.0121796

At 822.19 rpm the fan power = 0.0121796×822.19^2 = 8233.36 W

**At second point: I_a = 10.5 A & E_a = 393 at 600 rpm

With load at the same current; E_a = 500 – 10.5×3.5 = 463.25 V

$$\frac{393}{463.25} = \frac{600}{N} \rightarrow N = 707.252\, rpm$$

$P_{em} = E_a I_a$ = 463.25×10.5 = 4864.125 W

P_{out} = 4864.125 – 450 = 4414.125 W

The power required by the fan load $P_{fan} = K \times N^2$

At 707.252 rpm the fan power = 0.0121796×707.252^2 = 6092.3016 W

***At third point: I_a = 14 A & E_a = 434 at 600 rpm

With load at the same current; $E_a = 500 - 14 \times 3.5 = 451$ V

$$\frac{434}{451} = \frac{600}{N} \rightarrow N = 623.5023\ rpm$$

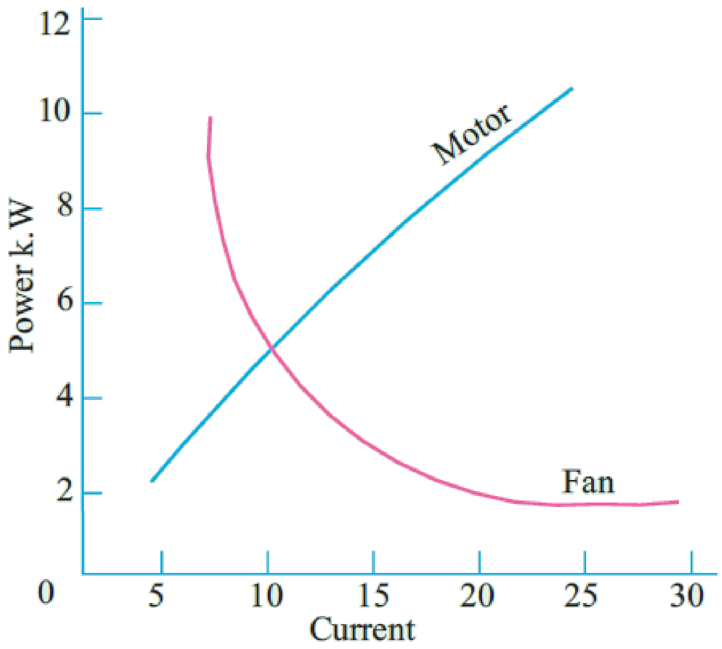
$P_{em} = E_a I_a = 451 \times 14 = 6314$ W

$P_{out} = 6314 - 450 = 5864$ W

The power required by the fan load $P_{fan} = K \times N^2$

At 707.252 rpm the fan power = $0.0121796 \times 623.5023^2 = 4734.8818$ W

After finishing all points, we can plot the relation between the motor output power vs current and the fan input power vs current as shown in figure below.



From the figure, the two curves intersect at a current of 12 A.

Then the input motor power = $500 \times 12 = 6000$ W

Starter of DC Motors

As we know, the armature current of a DC motor is given by

$$I_a = \frac{V_t - E_a}{R_a}$$

At starting, E_a is zero as the motor speed is zero, and the starting current will be high.

$$I_a|_{start} = \frac{V_t}{R_a}$$

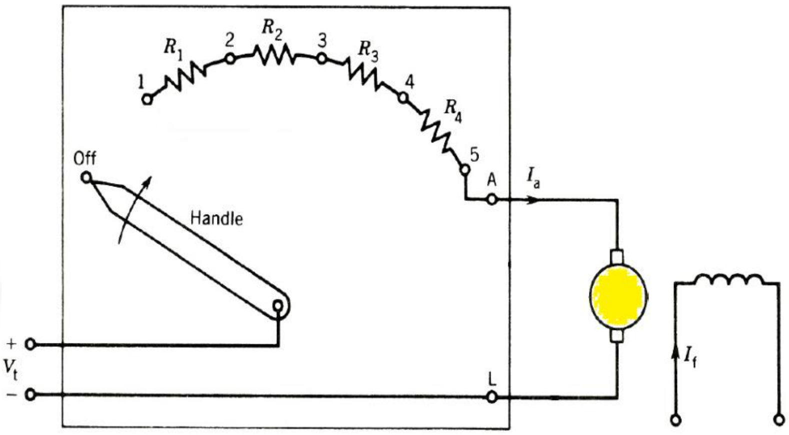
Since R_a is small, the starting current is very large. The starting current can be limited to a safe value by the following methods:

1. Insert an external resistance, $R_1 \rightarrow R_4$, at start as shown in figure below.
2. Use a low DC terminal voltage (V_t) at start. This, of course, requires a variable-voltage supply.

With an external resistance (R_{ae}) in the armature circuit, the armature current as the motor speeds up is given by:

$$I_a = \frac{V_t - E_a}{R_a + R_{ae}}$$

The back emf E_a increases as the speed increases. Therefore, the external resistance R_{ae} can be gradually taken out as the motor speeds up without the current exceeding a certain limit.



In the starter, shown above, at start, the handle is moved to position 1. All the resistances, R_1, R_2, R_3, and R_4, appear in series with the armature and thereby limit the starting current. As the motor speeds up, the handle is moved to positions 2, 3, 4, and finally 5. At position 5 all the resistances in the starter are taken out of the armature circuit.

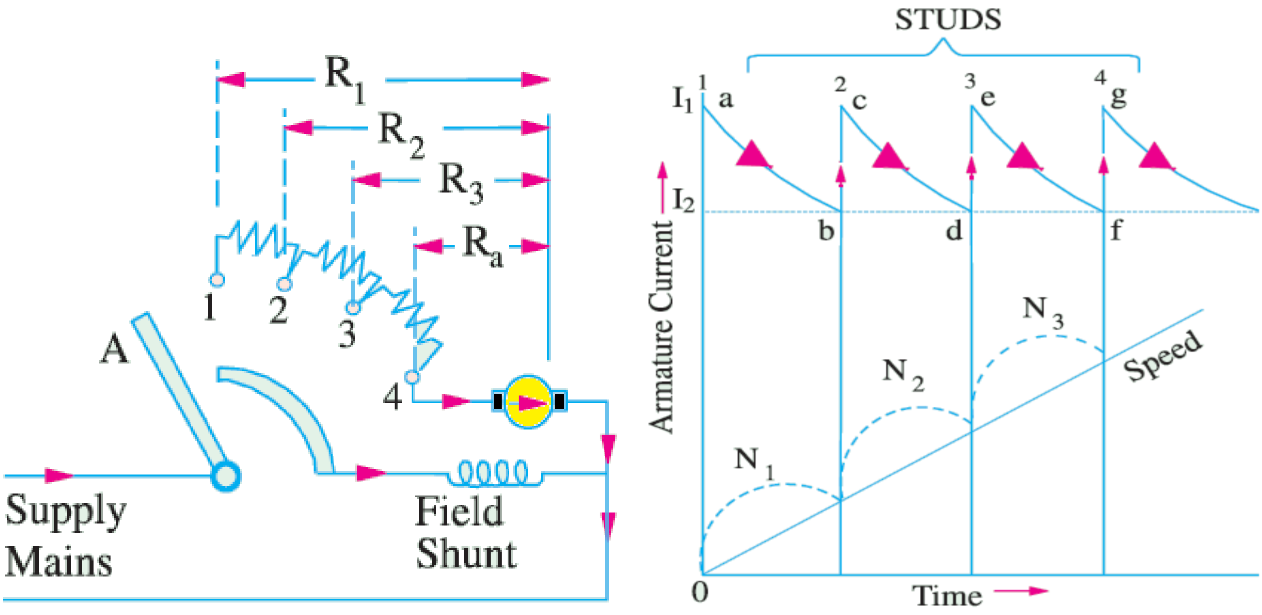
Shunt Motor Starter

It is usual to allow an overload of 50% at starting and to advance the starter a step when armature current has fallen to definite lower value I_2. Either this lower current limit may be fixed, or the number of starter steps may be fixed.

Assuming that the starter with 4 steps is connected to a shunt motor as shown below. When arm *A* contacts stud No. 1, at that point the motor speed = 0 and also $E_a = 0$

$$I_1 = \frac{V}{R_1}$$

Where I_1 is the maximum permissible current (usually limited to 1.5 times the full-load current of the motor), R_1 armature and starter resistance.



This current I_1 produce starting torque 1.5 times the full load torque. Therefore, the motor speed increases to N_1 and the back emf increases to E_{b1}. Consequently, the armature current falls to a predetermined value I_2 (also called I_{min}). At the time leaving stud No. 1

$$I_2 = \frac{V - E_{b1}}{R_1} \qquad (1)$$

Arm *A* is moved to stud No. 2. at that point the motor speed = N_1 and emf = E_{b1}, but the motor current jumped to I_1

$$I_1 = \frac{V - E_{b1}}{R_2} \qquad (2)$$

From (1) & (2)

$$\frac{I_1}{I_2} = \frac{R_1}{R_2}$$

Again, the motor speed increases to N_2 and the back emf increases to E_{b2}. Consequently, the armature current falls to I_2. At the time leaving stud No. 2

$$I_2 = \frac{V - E_{b2}}{R_2} \qquad (3)$$

Arm *A* is moved to stud No. 3, at that point the motor speed = N_2 and emf = E_{b2}, but the motor current jumped to I_1

$$I_1 = \frac{V - E_{b2}}{R_3} \qquad (4)$$

From (3) & (4)

$$\frac{I_1}{I_2} = \frac{R_2}{R_3}$$

Again, the motor speed increases to N_3 and the back emf increases to E_{b3}. Consequently, the armature current falls to I_2. At the time leaving stud No. 3

$$I_2 = \frac{V - E_{b3}}{R_3} \qquad (5)$$

Arm *A* is moved to stud No. 4, at that point the motor speed = N_3 and emf = E_{b3}, but the motor current jumped to I_1

$$I_1 = \frac{V - E_{b2}}{R_a} \qquad (6)$$

From (5) & (6)

$$\frac{I_1}{I_2} = \frac{R_3}{R_a}$$

From the above relations,

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_a} = K$$

We found that:

$$R_3 = K\, R_a$$

$$R_2 = K^2\, R_a$$

$$R_1 = K^3\, R_a$$

Therefore, for 4-steps starter, there are 3 resistances as indicated above.

Generally, for n-steps starter, there are n-1 resistances given as:

$$R_1 = K^{n-1} R_a$$

$$K^{n-1} = \frac{R_1}{R_a}$$

By replacing R_1 by V/I_1, then

$$(K)^{n-1} = \frac{V}{I_1 R_a}$$

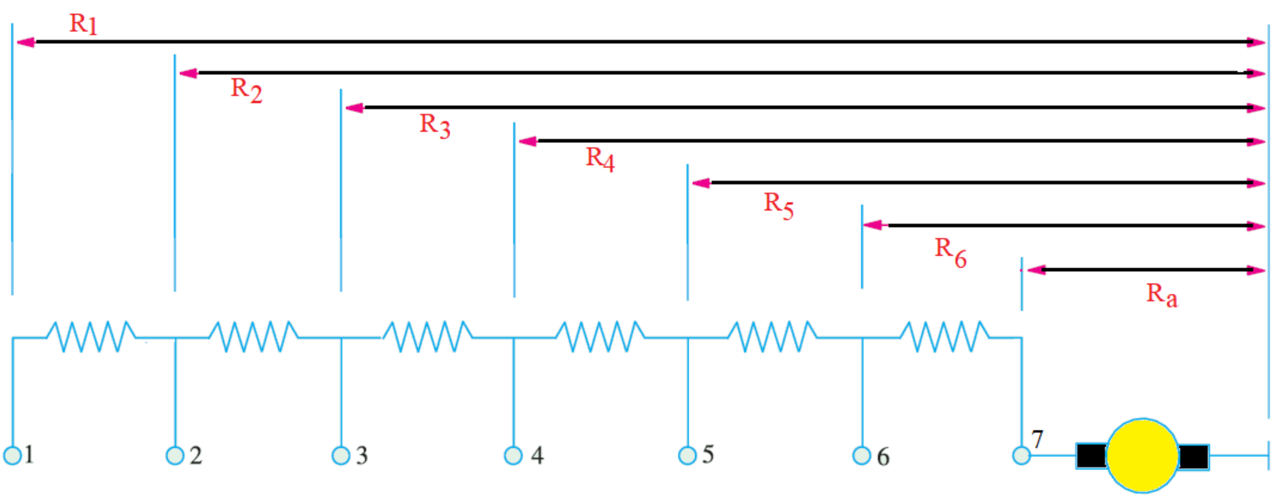
Also,

$$(K)^n = \frac{V}{I_1 R_a} \times K = \frac{V}{I_1 R_a} \times \frac{I_1}{I_2}$$

$$(K)^n = \frac{V}{I_2 R_a}$$

Example:

A 220-V shunt motor has an armature resistance of 0.4 Ω. The armature current at starting must not exceed 40 A. If the number of the starter sections is 6, calculate the values of the resistor steps to be used in this starter.



Since there are 6 sections → n = 7

Also I_1 = 40 A

$$I_1 = \frac{V}{R_1} \rightarrow R_1 = \frac{220}{40} = 5.5\ \Omega$$

$$K^{n-1} = \frac{R_1}{R_a} \rightarrow K^6 = \frac{5.5}{0.4} = 13.75$$

$6 \log K = \log 13.75 \rightarrow K = 1.5478$

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \frac{R_4}{R_5} = \frac{R_5}{R_6} = \frac{R_6}{R_a} = K$$

$$\frac{R_1}{R_2} = K \rightarrow R_2 = \frac{R_1}{K} = \frac{5.5}{1.5478} = 3.5534\ \Omega$$

$$\frac{R_2}{R_3} = K \rightarrow R_3 = \frac{R_2}{K} = \frac{3.5534}{1.5478} = 2.2958$$

$$\frac{R_3}{R_4} = K \rightarrow R_4 = \frac{R_3}{K} = \frac{2.2958}{1.5478} = 1.4833$$

$$\frac{R_4}{R_5} = K \rightarrow R_5 = \frac{R_4}{K} = \frac{1.4833}{1.5478} = 0.9583$$

$$\frac{R_5}{R_6} = K \rightarrow R_6 = \frac{R_5}{K} = \frac{0.9583}{1.5478} = 0.6191$$

Another Solution

$$R_6 = KR_a = 1.5478 \times 0.4 = 0.61912$$

$$R_5 = K^2R_a = 1.5478^2 \times 0.4 = 0.958274$$

$$R_4 = K^3R_a = 1.5478^3 \times 0.4 = 1.48322$$

$$R_3 = K^4R_a = 1.5478^4 \times 0.4 = 2.295722$$

$$R_2 = K^5R_a = 1.5478^5 \times 0.4 = 3.55332$$

$$R_1 = K^6R_a = 1.5478^6 \times 0.4 = 5.5$$

Resistance of 1st section = $R_1 - R_2 = 5.5 - 3.5534 = 1.9466\ \Omega$

Resistance of 2nd section = $R_2 - R_3 = 3.5534 - 2.2958 = 1.2576\ \Omega$

Resistance of 3ed section = $R_3 - R_4 = 2.2958 - 1.4833 = 0.8125\ \Omega$

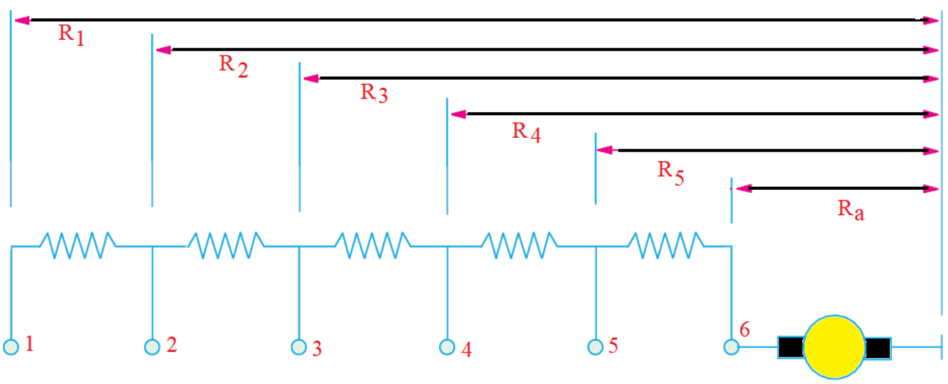
Resistance of 4th section = $R_4 - R_5 = 1.4833 - 0.9583 = 0.525\ \Omega$

Resistance of 5th section = $R_5 - R_6 = 0.9583 - 0.6191 = 0.3392\ \Omega$

Resistance of 6th section = $R_6 - R_a = 0.6191 - 0.4 = 0.2191\ \Omega$

Example:

Find the value of the step resistance in a 6-stud starter for a 5 HP, 200-V shunt motor. The maximum current in the line is limited to twice the full-load value. The armature Cu loss is 50% of the total loss. The normal field current is 0.6 A and the full-load efficiency is 88%.



The motor output power = 5×746 = 3730 W

Input power = P_{out}/η = 3730 / 0.88 = 4238.6364 W

Input power = $V_t \times I_t \rightarrow I_t$ = 4238.6364 / 200 = 21.1932 A

Armature Current (I_a) = $I_t - I_{sh}$ = 21.1932 – 0.6 = 20.5932 A

Total power loss = 4238.6364 – 3730 = 508.6364 W

$P_{cu\,a}$= 0.5×508.6364 = 254.3182 W

$P_{cu\,a}= I_a^2 R_a \rightarrow R_a$ = 254.3182 / 20.5932^2 = 0.5997 Ω

The starter has 6 studs → n = 6

Max line current is limited to twice the full-load value i.e 2×21.1932 = 42.3864 A

Max armature current I_1 = 42.3864 – 0.6 = 41.7864 A

$$I_1 = \frac{V}{R_1} \rightarrow R_1 = \frac{200}{41.7862} = 4.78625\ \Omega$$

$$K^{n-1} = \frac{R_1}{R_a} \rightarrow K^5 = \frac{4.78625}{0.5997} = 7.9811$$

5 log K = log 7.9811 → K = 1.515

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \frac{R_4}{R_5} = \frac{R_5}{R_a} = K$$

$$\frac{R_1}{R_2} = K \rightarrow R_2 = \frac{R_1}{K} = \frac{4.78625}{1.515} = 3.159241\ \Omega$$

$$\frac{R_2}{R_3} = K \rightarrow R_3 = \frac{R_2}{K} = \frac{3.159241}{1.515} = 2.08531$$

$$\frac{R_3}{R_4} = K \rightarrow R_4 = \frac{R_3}{K} = \frac{2.08531}{1.515} = 1.376441$$

$$\frac{R_4}{R_5} = K \rightarrow R_5 = \frac{R_4}{K} = \frac{1.376441}{1.515} = 0.908542$$

Resistance of 1st section = $R_1 - R_2$ = 4.78625 – 3.159241 = 1.627 Ω

Resistance of 2nd section = $R_2 - R_3$ = 3.159241 – 2.08531 = 1.07393 Ω

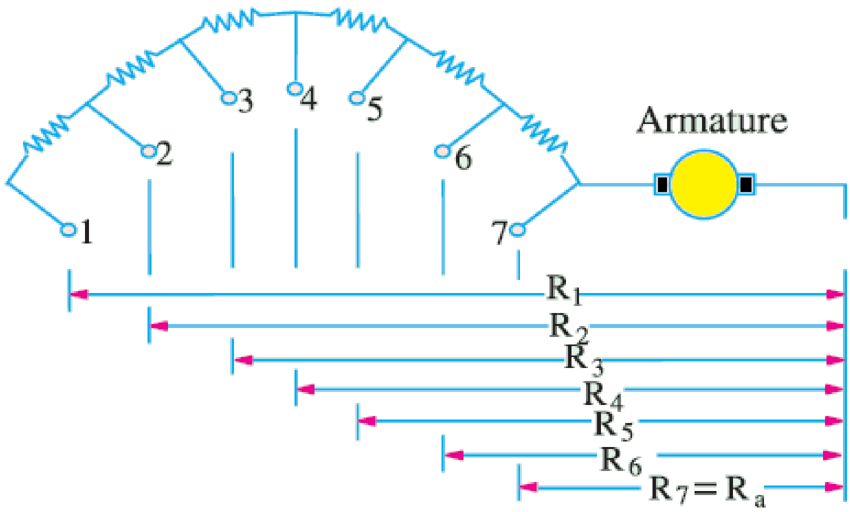
Resistance of 3^{ed} section = $R_3 - R_4$ = 2.08531 – 1.376441 = 0.70887 Ω

Resistance of 4th section = $R_4 - R_5$ = 1.376441 – 0.908542 = 0.4679 Ω

Resistance of 5th section = $R_5 - R_a$ = 0.908542 – 0.5997 = 0.308842 Ω

Example:

Design the resistance sections of a seven-stud starter for 36.775 kW, 400V, DC shunt motor. Full-load efficiency is 92%, total Cu losses are 5% of the input. Shunt field resistance is 200 Ω. The lower limit of the current through the armature is to be full-load value.



$$\text{Output} = 36{,}775 \text{ W}; \quad \text{Input} = 36{,}775/0.92 = 39{,}980 \text{ W}$$

$$\text{Total Cu loss} = 0.05 \times 39{,}980 = 1{,}999\text{W}$$

$$\text{Shunt Cu loss} = V^2/R_{sh} = 400^2/200 = 800 \text{ W}$$

$$\text{Armature Cu loss} = 1{,}999 - 800 = 1199 \text{ W}$$

$$\text{F.L. input current} = 39{,}980/400 = 99.95 \text{ A}$$

$$I_{sh} = 400/200 = 2\text{A}; \quad I_a = 99.95 - 2 = 97.95 \text{ A}$$

$$\therefore \quad 97.95^2 R_a = 1199 \text{ W} \quad \text{or} \quad R_a = 0.125\ \Omega$$

Now, minimum armature current equals full-load current *i.e.* $I_a = 97.95$ A.

$$\text{or} \quad K^7 = 400/97.95 \times 0.125 = 32.68$$

$$\therefore \quad K = 32.68^{1/7} = 1.645$$

$$I_1 = \text{maximum permissible armature current}$$

$$= KI_2 = 1.645 \times 97.94 = 161 \text{ A}$$

$$\therefore \quad R_1 = V/I_1 = 400/161 = 2.483\ \Omega$$

$$R_2 = R_1/K = 2.483/1.645 = 1.51\ \Omega$$

$$R_3 = 1.51/1.645 = 0.917\ \Omega$$

$$R_4 = 0.917/1.645 = 0.557\ \Omega$$

$$R_5 = 0.557/1.645 = 0.339\ \Omega$$

$$R_6 = 0.339/1.645 = 0.206\ \Omega$$
$$R_7 = 0.206/1.645 = 0.125\ \Omega$$

$$\text{Resistance in 1st step} = R_1 - R_2 = \mathbf{0.973\ \Omega}$$
$$\text{Resistance in 2nd step} = R_2 - R_3 = \mathbf{0.593\ \Omega}$$
$$\text{Resistance in 3rd step} = R_3 - R_4 = \mathbf{0.36\ \Omega}$$
$$\text{Resistance in 4th step} = R_4 - R_5 = \mathbf{0.218\ \Omega}$$
$$\text{Resistance in 5th step} = R_5 - R_6 = \mathbf{0.133\ \Omega}$$
$$\text{Resistance in 6th step} = R_6 - R_a = \mathbf{0.081\ \Omega}$$

Example

Calculate the resistance steps for the starter of a 250-V, DC shunt motor having an armature resistance of 0.125 Ω and a full-load current of 150 A. The motor is to start against full-load and maximum current is not to exceed 200 A

As the motor is to start against its full-load, the minimum current is its F.L. current *i.e.* 150A. Here I_1 = 200 A; I_2 = 150 A; R_1 = 250/200 = **1.25** Ω

$(I_1/I_2)^{n-1} = R_1 / R_a$

$$\therefore\ (200/150)^{n-1} = 1.25/0.125 = 10 \quad \text{or} \quad (4/3)^{n-1} = 10$$
$$(n-1)\log 4/3 = \log 10 \quad \text{or} \quad (n-1) \times 0.1249 = 1$$
$$\therefore\ (n-1) = 1/0.1249 = 8$$

Hence, there are 9 studs and 8 steps

$$R_2 = R_1 \times I_2/I_1 = 1.25 \times 3/4 = 0.938\ \Omega$$
$$R_3 = 0.938 \times 3/4 = 0.703\ \Omega$$
$$R_4 = 0.703 \times 3/4 = 0.527\ \Omega$$
$$R_5 = 0.527 \times 3/4 = 0.395\ \Omega$$
$$R_6 = 0.395 \times 3/4 = 0.296\ \Omega$$
$$R_7 = 0.296 \times 3/4 = 0.222\ \Omega$$
$$R_8 = 0.222 \times 3/4 = 0.167\ \Omega$$
$$R_a = 0.167 \times 3/4 = 0.125\ \Omega$$

Resistance of 1st element	=	1.25 – 0.938	= **0.312 Ω**
,, 2nd ,,	=	0.938 – 0.703	= **0.235 Ω**
,, 3rd ,,	=	0.703 – 0.527	= **0.176 Ω**
,, 4th ,,	=	0.527 – 0.395	= **0.132 Ω**
,, 5th ,,	=	0.395 – 0.296	= **0.099 Ω**
,, 6th ,,	=	0.296 – 0.222	= **0.074 Ω**
,, 7th ,,	=	0.222 – 0.167	= **0.055 Ω**
,, 8th ,,	=	0.167 – 0.125	= **0.042 Ω**

Example:

4-pole, lap-wound armature winding of a 500-V, DC shunt motor is housed in 60 slots each slot containing 20 conductors. The armature resistance is 1.31 Ω. If during the period of starting, the minimum torque is required to be 218 N.m and the maximum torque 1.5 times the minimum torque, find out how many sections the starter should have and calculate the resistances of these sections. Take the useful flux per pole to be 23 mWb.

Since lap winding, then $a = 2P$, $Z = 60\times20 = 1200$

$K_a = 2P\times Z/(2\pi\times a) = Z/2\pi = 1200/2\pi = 190.985932$

$T = K_a \times \Phi \times I_a \rightarrow 218 = 190.985932 \times 0.023 \times I_a \rightarrow I_a = 49.6281$ A

Therefore the minimum current $I_2 = 49.6281$ A

The maximum current $I_1 = 1.5\times I_2 = 74.4421$ A

$K = I_1 / I_2 = 74.4421 / 49.6281 = 1.5$

$R_1 = V / I_1 = 500 / 74.4421 = 6.71663\ \Omega$

$$K^{n-1} = \frac{R_1}{R_a} \rightarrow 1.5^{n-1} = \frac{6.71663}{1.31} = 5.127198$$

$$(n-1)\log 1.5 = \log 5.127198 \qquad \rightarrow \quad n = 5.03132$$

Therefore $n = 5$

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \frac{R_4}{R_a} = K$$

$$\frac{R_1}{R_2} = K \rightarrow R_2 = \frac{R_1}{K} = \frac{6.71663}{1.5} = 4.4778\ \Omega$$

$$\frac{R_2}{R_3} = K \quad \rightarrow \quad R_3 = \frac{R_2}{K} = \frac{4.4778}{1.5} = 2.9852$$

$$\frac{R_3}{R_4} = K \quad \rightarrow \quad R_4 = \frac{R_3}{K} = \frac{2.9852}{1.5} = 1.9901$$

Resistance of 1st section = $R_1 - R_2$ = 6.71663 – 4.4778 = 2.23883 Ω

Resistance of 2nd section = $R_2 - R_3$ = 4.4778 – 2.9852 = 1.4926 Ω

Resistance of 3^{ed} section = $R_3 - R_4$ = 2.9852 – 1.9901 = 0.9951 Ω

Resistance of 4th section = $R_4 - R_a$ = 1.9901 – 1.31 = 0.6801 Ω

Example:

4-pole, lap-wound armature winding of a 31.5 HP, 250 V, DC shunt motor having 60 slots each slot containing 20 conductors, the useful flux per pole to be 7.5922 mWb. The full-load electromagnetic torque is 145 N.m. The shunt field resistance is 125Ω. The total cupper losses are 53% of the total losses. A starter with *n*-studs is used to start this motor. The lower limit and upper limit of armature current are to be full-load value and 188.290984% of the full load value.

Calculate:

a) The value of *n*,
b) Resistance of each section of the starter.

The motor output power = 31.5×746 = 23499 W

I_{sh} = 250 / 125 = 2 A

$P_{cu\ sh} = 2^2 \times 125 = 500$ W

Since lap winding, then a = 2P, Z = 60×20 = 1200

$K_a = 2P \times Z/(2\pi \times a) = Z/2\pi = 1200/2\pi = 190.985932$

$T = K_a \times \Phi \times I_a \rightarrow 145 = 190.985932 \times 0.0075922 \times I_a \rightarrow I_a = 100$ A

Therefore the minimum current I_2 = 100 A

The maximum current $I_1 = 1.88290984 \times I_2 = 188.290984$ A

$K = I_1 / I_2 = 188.290984 / 100 = 1.88290984$

$I_t = I_a + I_{sh} = 100 + 2 = 102$ A

$P_{in} = V_t I_t = 250 \times 102 = 25500$ W

Total losses = 25500 – 23499 = 2001 W

Total Cu losses = 0.53 × 2001 = 1060.53 W

Armature Cu loss = 1060.53 – 500 = 560.53 W

$R_a = 560.53 / 10000 = 0.056053\ \Omega$

$R_1 = V / I_1 = 250 / 188.290984 = 1.327732187\ \Omega$

$$K^{n-1} = \frac{R_1}{R_a} \rightarrow 1.88290984^{n-1} = \frac{1.327732187}{0.056053} = 23.68708521$$

$$(n-1)\ \log 1.88290984 = \log 23.68708521 \qquad \rightarrow \quad n = 6$$

Therefore $n = 6$ ##

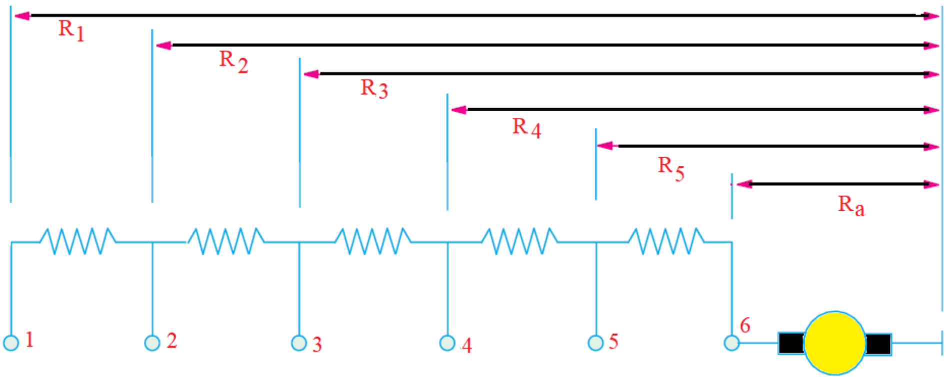
$$R_i = K^{n-i} R_a$$

$$R_2 = (1.88290984)^{6-2} 0.056053 = 0.704558\ \Omega$$

$$R_3 = (1.88290984)^{6-3} 0.056053 = 0.374186\ \Omega$$

$$R_4 = (1.88290984)^{6-4} 0.056053 = 0.1987275\ \Omega$$

$$R_5 = (1.88290984)^{6-5} 0.056053 = 0.10554275\ \Omega$$



Resistance of 1st section = $R_1 - R_2$ = 1.327732187 – 0.704558 = 0.623174187 Ω

Resistance of 2nd section = $R_2 - R_3$ = 0.704558 − 0.374186 =0.330372 Ω

Resistance of 3ed section = $R_3 - R_4$ = 0.374186 − 0.1987275 = 0.1754585 Ω

Resistance of 4th section = $R_4 - R_5$ = 0.1987275 − 0.10554275 = 0.09318475 Ω

Resistance of 5th section = $R_5 - R_a$ = 0.10554275 − 0.056053= 0.04948975 Ω

SHEET (3) DC Machines

Part (A) DC Generators

Problem (1):

Draw the developed winding diagram and write down the winding table for a 2-layer simplex lap-winding for a 4-pole DC generator having 20 slots. What are the back and front pitches as measured in terms of armature conductors?

Problem (2):

A 4-pole 250-V, DC series motor has a wave-wound armature with 496 conductors. The value of flux per pole under these conditions is 22 mWb and the corresponding iron, friction and windage losses total 810 W. Armature resistance = 0.19 Ω, field resistance = 0.14 Ω. Calculate

(*a*) the gross torque

(*b*) the speed

(*c*) the output torque and

(*d*) the efficiency, if the motor current is 50 A

Problem (3):

A 230 V series motor is taking 50 A. Resistance of armature and series field windings is 0.2 Ω and 0.1 Ω respectively. Calculate:

(*a*) brush voltage

(*b*) back e.m.f.

(*c*) power wasted in armature

(*d*) mechanical power developed

Problem (4):

A DC series motor on full-load takes 50 A from 230 V DC mains. The total resistance of the motor is 0.22 Ω. If the iron and friction losses together amount to 5% of the input, calculate the power delivered by the motor shaft. Total voltage drop due to the brush contact is 2 A.

Problem (5):

A 2-pole DC shunt motor operating from a 200 V supply takes a full-load current of 35 A, the no-load current being 2 A. The field resistance is 500 Ω and the armature has a resistance of 0.6 Ω. Calculate the efficiency of the motor on full-load. Take the brush drop as being equal to 1.5 V per brush arm.

Problem (6):

A DC shunt generator running at 850 r.p.m. gave the followig *O.C.C.* data:

Field current (A)	:	0	0.5	1	2	3	4	5
E.M.F. (V)	:	10	60	120	199	232	248	258

If the resistance of the shunt field is 50 Ω, determine the additional resistance required in the shunt field circuit to give 240 V at a speed of 1000 r.p.m.

Problem (7):

The open-circuit characteristic of a separately-excited DC generator driven at 1000 r.p.m. is as follows:

Field current :	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
E.M.F. volts :	30.0	55.0	75.0	90.0	100.0	110.0	115.0	120.0

If the machine is connected as shunt generator and driven at 1000 r.p.m. and has a field resistance of 100 Ω, find

(a) open-circuit voltage and exciting current

(b) the critical resistance and

(c) resistance to induce 115 volts on open circuit

Problem (8):

The O.C.C. of a DC generator driven at 400 rev/min is as follows:

Field current (A)	:	2	3	4	5	6	7	8	9
Terminal volts	:	110	155	186	212	230	246	260	271

Find :

(a) voltage to which the machine will excite when run as a shunt generator at 400 rpm with shunt field resistance equal to 34 Ω.

(b) resistance of shunt circuit to reduce the O.C. voltage to 220 V.

(c) critical value of the shunt field circuit resistance.

Part (B) DC Motors

Problem (9):

The resistance of the armature of a 250-V shunt motor is 0.3 Ω and its full-load speed is 1000 r.p.m. Calculate the resistance to be inserted in series with the armature to reduce the speed with full-load torque to 800 r.p.m., the full-load armature current being 5A. If the load torque is then halved, at what speed will the motor run?

Problem (10):

A 500-V DC shunt motor draws a line-current of 5 A on light-load and running at 600 rpm. If the armature resistance is 0.15 Ω and the field resistance is 200 Ω, determine

a) Efficiency of the machine running as a generator delivering a load current of 40 A.
b) At what speed should the generator be run, if the shunt-field is not changed

Problem (11):

A 460-V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and % change in torque if the load is reduced so that the motor is taking 30 A. Giving that total resistance of the armature and field circuits is 0.8 Ω.

Problem (12):

The following results were obtained from a static torque test on a DC series motor:

Current (A)	20	30	40	50
Torque (N.m)	128.8	230.5	349.8	469.2

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V. Resistance of armature and field winding is 0.5 Ω. Ignore iron and friction losses.

Problem (12):

A fan which requires 8 HP at 700 rpm is coupled directly to a DC series motor. Calculate the input to the motor when the supply voltage is 500 V, assuming that power required for fan varies as the square of the speed. For the purpose of obtaining the magnetisation characteristics, the motor was running as a self-excited generator at 600 r.p.m. and the relationship between the terminal voltage and the load current was found to be as:

Current (A)	7	10.5	14	27.5
No-Load Voltage (V)	347	393	434	458

The resistance of both the armature and field windings of the motor is 3.5 Ω and the core, friction and other losses may be assumed to be constant at 450 W for the speeds corresponding to the above range of currents at normal voltage.

Problem (13):

Design the resistance sections of a seven-stud starter for 36.775 kW, 400V, DC shunt motor. Full-load efficiency is 92%, total Cu losses are 5% of the input. Shunt field resistance is 200 Ω. The lower limit of the current through the armature is to be full-load value.

Problem (14):

4-pole, lap-wound armature winding of a 500-V, DC shunt motor is housed in a 60 slots each slot containing 20 conductors. The armature resistance is 1.31 Ω. If during the period of starting, the minimum torque is required to be 218 N.m and the maximum torque 1.5 times the minimum torque, find out how many sections the starter should have and calculate the resistances of these sections. Take the useful flux per pole to be 23 mWb.